The Effects of Stock Returns on Consumption : Evidence from Nonlinearities

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Abstract

This study explains nonlinearities and cyclical movements of Korean stock returns and consumption using a STAR (Smooth Transition Autoregressive) model. According to nonlinear tests, both time series do not reject nonlinearities and show cyclical movements. Also a nonlinear Granger causality test finds that stock returns affect consumption.

JEL Classification: C12, C13, C32, E21, G10

Key words : STAR model, nonlinearity, wealth effect, stock, consumption

* The author acknowledges financial support from「Ewha Womans university」for this study.

I. Introduction

This study examines nonlinearities and dynamic properties of Korean stock returns and consumption time series using a STAR (Smooth Transition Autoregressive) model. Then we extend the empirical results with the nonlinear relationship between consumption and stock returns. Economic theory supports the nonlinearities and cycles of both stock and consumption data. For example, Brock and LeBaron (1996)'s 'heterogeneous belief', Peters (1994)'s 'time varying investment horizon and heterogeneity in the risk', and Lux (1995)'s 'herd behavior' represent sources of nonlinearities of above time series. Also many recent econometric studies have investigated the nonlinearities of economic data.[1)] Most of this nonlinear research, though, relates to macro variables such as GDP, industrial production, and unemployment and we cannot find any previous studies of Korean stocks and consumption.

We use a STAR specification for modeling nonlinearity. This model was originally developed by Teräsvirta and Anderson (1992) for the nonlinearity of industrial production. The major reason why we use this STAR specification is that it explains the 'smooth transition of regimes'. This is a particular merit of the STAR model compared to other nonlinear models such as the threshold autoregressive (TAR) model and the Markov regime switching model,which explain instant regime switching. As mentioned above, we believe that the macro variables in our study will show a smooth regime transition.[2)]

In the last part of this work, we study stock returns' Granger causality on consumption. We find quite a few research papers on this issue. For example, Lee (1994) and Jung(2003) study asset prices' effects on consumption by estimating bubbles in stock market. Choi and Lee (1991), Kim and Moon (2001), and Yoon (2002) explore the same topic with asset price volatility.

In Section 2, we introduce STAR model estimation. Section 3 analyzes nonlinearity tests and estimation. Section 4 shows the dynamic properties of each time series based on the empirical results of Section 3. Section 5 performs linear and nonlinear Granger causality between stock returns and consumption. Section 6 concludes this study.

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1) See Hsieh (1991), De Lima (1994), and Ryden, Tersvirta, and Asbrink (1998).
2) See Sarantis (2001).

II. Specification and estimation of STAR models

The STAR model of order k, for variable s_t, is defined as

$$s_t = \beta_0 + \beta_1 x_t \, (\theta_0 + \theta_1 x_t) F(s_{t-d}) + u_t \tag{1}$$

where $x_t = (s_{t-1}, s_{t-2}, \cdots, s_{t-k})'$, $\beta_1 = (\beta_1, \beta_2, \cdots, \beta_k)'$, $\theta_1 = (\theta_1, \theta_2, \cdots, \theta_k)'$, $u_t \sim$ nid$(0, \sigma^2)$, $F(..)$ is the transition function, s_{t-d} is the transition variable, and d is the delay parameter. In this model, nonlinearities arise through conditioning on lagged stock returns.

Following previous studies, the transition function, $F(..)$, is assumed to be either logistic function

$$F(s_{t-d}) = [1 + \exp\{-\gamma \cdot (s_{t-d} - c)\}]^{-1}, \ \gamma > 0 \tag{2}$$

or an exponential function

$$F(s_{t-d}) = 1 - \exp\{-\gamma \cdot (s_{t-d} - c)^2\} \tag{3}$$

where γ measures the speed of transition from one regime to the other, and c indicates the half-way point between the two regimes.

Eq. (1) with transition function (2) is referred to as the logistic STAR (hereafter LSTAR(p)) model, whereas Eq. (1) combined with transition function (3) defines the exponential STAR (hereafter ESTAR(p)) model. According to STAR models, expansion and contraction represent two distinct economic phases, but the transition between the two regimes is smooth, governed by s_{t-d}. The LSTAR and ESTAR models, however, describe quite different types of dynamic behavior. The LSTAR model allows the expansion and contraction regimes to have different dynamics, with the transition from one to the other being smooth. This implies that the LSTAR model can characterize asymmetric cycles in stock returns. On the other hand, the ESTAR model suggests that the two regimes have rather similar dynamics, while the behavior of stock returns in the transition period (middle regime) can be different. Hence the ESTAR model is also capable of describing asymmetries in stock returns.

The modeling procedure for building STAR models is carried out in three stages (see Granger and Terasvirta, 1993, pp. 113-124; Terasvirta, 1994; Eitrheim and Terasvirta, 1996).

(a) Specify a linear AR (autoregressive) model. We estimate AR models of different orders and the maximum value of p^* is chosen on the basis of the AIC criterion and the Ljung-Box statistics for autocorrelation.

(b) Test linearity against STAR models, for different values of the delay parameter d, using the linear model specified in (a) as null. To carry out this test, we estimate the auxiliary regression

$$s_t = \beta_0 + \beta_1 x_t + \beta_2 x_t \cdot s_{t-d} + \beta_3 x_t \cdot s_{t-d}^2 + \beta_4 x_t \cdot s_{t-d}^3 + w_t \tag{4}$$

The linearity test is $H_0 : \beta_2 = \beta_3 = \beta_4 = 0$. To specify the value of the delay parameter d, the estimation of (4) is carried out for a wide range of values, $1 \leq d \leq$ D. In cases where linearity is rejected for more than one value of d, then d is chosen by d = arg min p(d) for $1 \leq$ d $\leq$ D, where p(d) is the P-value of the linearity test (see Terasvirta and Anderson, 1992; Terasvirta, 1994).

(c) Choose between LSTAR and ESTAR models for those stock returns where linearity is rejected. This is done by applying the sequence of nested tests,

$$H_{04} : \beta_4 = 0 \tag{5}$$
$$H_{03} : \beta_3 = 0 / \beta_4 = 0 \tag{6}$$
$$H_{02} : \beta_2 = 0 / \beta_3 = \beta_4 = 0 \tag{7}$$

Rejection of H_{04} implies selecting the LSTAR model. If we accept H_{04} and reject H_{03}, we choose the ESTAR model. Accepting H_{04} and H_{03} and rejecting H_{02} leads to an LSTAR model. Granger and Terasvirta (1993), Terasvirta (1994), and Eitrheim and Terasvirta (1996) argue that strict application of this sequence of tests may lead to wrong conclusions, because the higher order terms of the Taylor expansion used in deriving these tests are disregarded. They therefore recommend the computation ofthe P-values for all F-tests of (5)-(7) and the choice of the STAR model on the basis of the lowest P-value; i.e. if the rejection of H_{03} is the strongest one, choose an ESTAR model, otherwise select an LSTAR model.

III. Empirical results

1. Data

We use two monthly data series of seasonally-adjusted real consumption growth and real stock returns over 1987.1 to 2003.1. Since monthly data supplies a relatively large sample size compared to yearly and quarterly data, this provides us with sufficient degrees of freedomfor a test of nonlinearity.

The monthly data on economic time series are known to be noisy. This would cause a serious problem with volatile stock market data. Therefore we use the annual growth rate of the data (the growth rate from the same month in the previous year).[3)]

For representative real stock data, we adjust KOSPI with dividends against the CPI growth rate. However, we were unable to obtain monthly consumption data from any Korean data archive. Therefore as an efficient proxy variable for consumption data, we employ the monthly sales index adjusted for inflation.

Basic statistics for the two data sets are summarized in [Table 1] and [Table 2]. And the graphs are shown in [Figure 1] and [Figure 2].

Table 1 Statistics summary of real stock returns and real consumption growth

variable	period	Number of Observation	Mean (%)	Standard Deviation (%)
real stock returns	1987.01-2003.01	193	12.2648	59.5541
real consumption growth	1987.01-2003.01	193	7.3925	6.5557

Table 2 Unit Root Test

variable	ADF	
	t statistics	P value
real stock returns	3.2229	0.0202
real consumption growth	3.0543	0.0318

3) See Neftci (1994), Tersvirta and Anderson (1992), McQueen and Thorley (1993), Estrella and Mishkin (1998), Birchenhall, Jessen, Osborn, and Simpson (1999), Skalin and Terasvirta (2000) for a similar transformation of various macro time series.

Figure 1 Real Stock Returns(%)

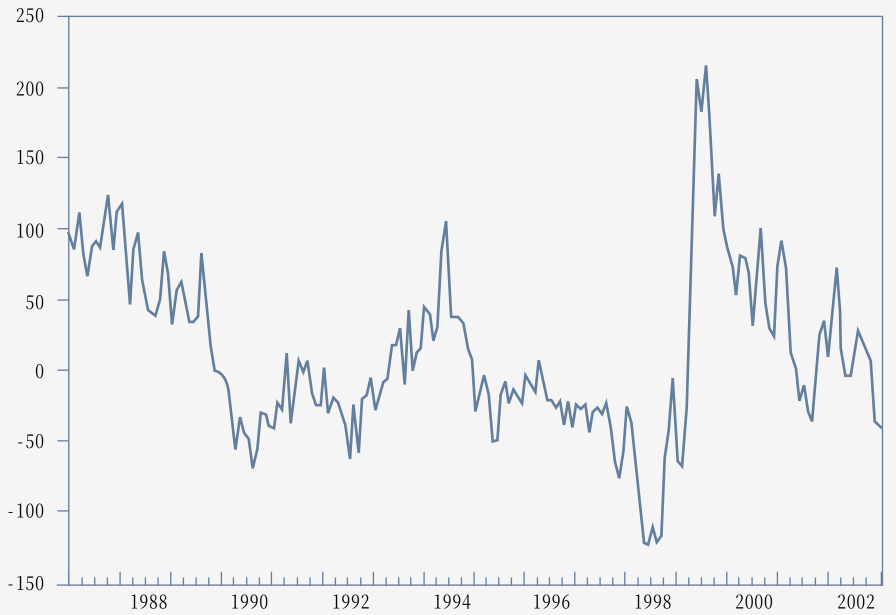
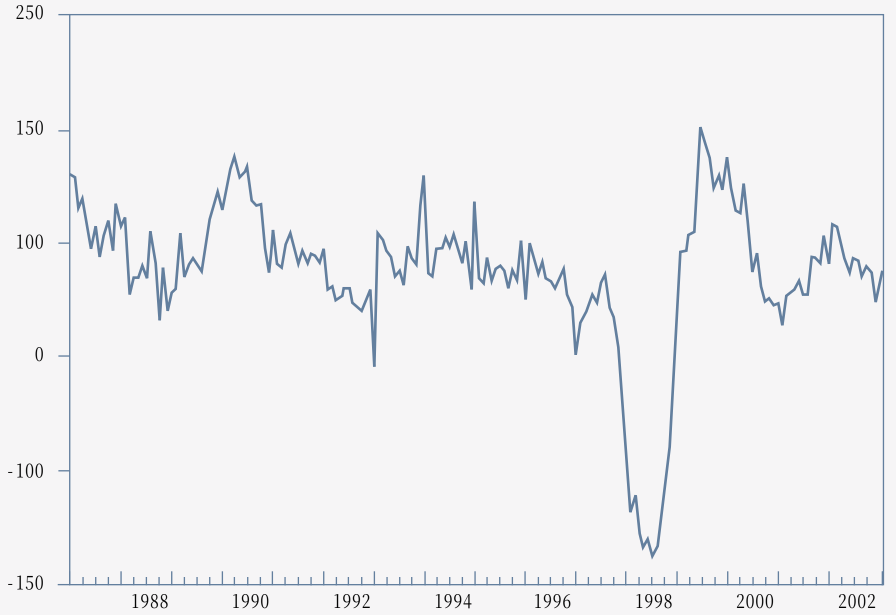


Figure 2 Real Consumption Growth Rate (%)



2. Nonlinearity Test and Decision of STAR model

The results for the linearity tests, together with the maximum lag of AR models and the Ljung-Box statistic (LB) for autocorrelation are shown in Table 3. The LB(..) statistics indicate that all AR models have white noise residuals. In carrying out linearity tests we have considered values for the delay parameter *d* over the range $1 \leq d \leq 10$, and then estimated the model and calculated the P-values for the linearity test for each value of *d*. The estimate of *d* is chosen by the lowest P-value.

The null hypothesis of linearity is rejected at the 5% level of significance for both consumption and stock return data.The value of the delay parameter varies across countries as shown in [Table 3], with stock returns having a delay parameter of six while real consumption has a delay of one.

Selection of LSTAR and ESTAR models is based on the tests reported in [Table 4]. The result suggests that the use of the ESTAR model for both variables. Hence the tests imply that symmetric expansion and contraction phases of stock and consumption data are likely.

Table 3 Unit Root Test

Delay (d)	real stock returns $p^* = 13$	real consumption growth rates $p^* = 8$
1	0.0002	0.0008*
2	0.0305	0.0016
3	0.0084	0.1479
4	0.0005	0.4040
5	0.0086	0.5124
6	0.0001*	0.5547
7	0.0012	0.6012
8	0.0003	0.6175
9	0.0021	0.7245
10	0.0015	0.7725

*means the maximum F statistics over $1 \leq d \leq 10$.

Table 4 Decision of STAR model

variable	Delay d^*	H_{04}: $\beta_4 = 0$	H_{03}: $\beta_3 = 0$ / $\beta_4 = 0$	H_{04}: $\beta_2 = 0$ / $\beta_3 = \beta_4 = 0$	Type of model
real stock returns	6	0.0515	0.0310	0.0554	ESTAR
real consumption growth rate	1	0.2099	0.0191	0.0034	ESTAR

Value in the table are P values for H_{04}, H_{03}, and H_{04}.

3. Estimation of STAR model

We follow the method of nonlinear least squares for estimation of STAR model. Estimates of parameters are shown in [Table 5].

Estimation of the transition parameter γ tends to be problematic (see Granger and Terasvirta, 1993; Terasvirta, 1994). Following the authors' suggestion, we have standardized the exponent of the transition function F(..) to make γ scale-free and thus easier to interpret.

In our estimation, a critical parameter in the STAR models is the transition coefficient γ. The estimates of γ have positive sign and are strongly significant (less than 1% level) in both variables. One interesting observation is the relatively small value of γ in variables.[4] This evidence suggests a slow transition from one regime to the other, contrary to the Markov regime-switching and the TAR models which assume a sharp switch between the two regimes. Parameter c indicates the halfway point between the expansion and contraction cyclical phases of stock returns. Real consumption's estimates of c is statistically significant at the 1% level but real stock's c estimate is not significant.

Table 5 Estimation of STAR Model

1. real stock returns (ESTAR)

$$r_t = a_0 + a_0 r_{t-2} + a_4 r_{t-4} + a_5 r_{t-5}$$
$$+ (b_0 + b_1 r_{t-1} + b_2 r_{t-2} + b_4 r_{t-4} + b_5 r_{t-5}) \cdot \left[\frac{1}{1 + \exp[-\frac{\gamma}{s.d.(r_t)}(r_{t-1} - c)]}\right]$$

	Estimate	s.d.	t stat	P value
a_0	10.1281	3.8499	2.6307	0.0093
a_2	1.8642	0.6710	2.7781	0.0061
a_4	-3.9178	1.6003	-2.4481	0.0153
a_5	1.5896	1.3091	1.2142	0.2263
b_0	-9.7626	3.8693	-2.5230	0.0125
b_1	0.7684	0.0915	8.3945	0.0000
b_2	-1.5988	0.6968	-2.2944	0.0229
b_4	3.9308	1.6077	2.4449	0.0155
b_5	-1.7106	1.3226	-1.2933	0.1976
γ	0.7379	0.2335	3.1592	0.0019
c	-0.1166	0.2748	-0.4245	0.6717
R^2	0.8726	Adjusted R^2		0.8654

4) This implies that the STAR specification was an effective model in explaining consumption and stock returns.

2. real consumption growth rates (ESTAR)

$$r_t = a_0 + a_1 r_{t-1} + a_4 r_{t-4} + a_5 r_{t-5} + a_6 r_{t-6} + a_9 r_{t-9} + a_{11} r_{t-11} + a_{12} r_{t-12} + a_{13} r_{t-13}$$
$$+ (b_0 + b_4 r_{t-4} + b_5 r_{t-5} + b_6 r_{t-6} + b_9 r_{t-9} + b_{11} r_{t-11}$$
$$+ b_{12} r_{t-12}) \cdot \left[\frac{1}{1 + \exp[-\frac{\gamma}{s.d.(r_t)}(r_{t-6} - c)]} \right]$$

	Estimate	s.d.	t stat	P value
a_1	-28.0482	4.3953	-6.3813	0.0000
a_1	0.8557	0.0576	14.8516	0.0000
a_4	-0.5181	0.1816	-2.8528	0.0049
a_5	-0.8428	0.2064	-4.0828	0.0001
a_6	-2.1648	1.1029	-1.9627	0.0514
a_9	1.0067	0.1322	7.6151	0.0000
a_{11}	0.5827	0.1249	4.6657	0.0000
a_{12}	-2.2503	0.2719	-8.2761	0.0000
a_{13}	0.2523	0.0887	2.8421	0.0051
b_0	27.923	4.8667	5.7377	0.0000
b_4	0.5869	0.2188	2.6816	0.0081
b_5	0.9358	0.2709	3.4538	0.0007
b_6	1.9962	1.1170	1.7870	0.0758
b_9	-0.8949	0.1724	-5.1885	0.0000
b_{11}	-0.6608	0.1650	-4.0031	0.0001
b_{12}	2.0089	0.2417	8.3092	0.0000
γ	1.4812	0.3436	4.3109	0.0000
c	-2.2648	0.4392	-5.1558	0.0000
R^2	0.8566	Adjusted R^2		0.8415

IV. Dynamic behavior

The dynamic behavior of the estimated nonlinear models can be investigated by examining the characteristic roots of the models, which are computed from the characteristic polynomial

$$\lambda^k - \sum_{j=1}^{k}(\beta_j + \theta_j F)\lambda^{k-j} = 0 \qquad (8)$$

Following previous researchers (see, for example, Terasvirta and Anderson, 1992; Terasvirta, 1994; Skalin and Terasviata, 1999), we calculate the roots for two regimes: firstly, $F = 0$, which corresponds to the lower (contraction) regime in the LSTAR model and the middle regime in the ESTAR model. Secondly, $F = 1$, which describes the upper (expansion) regime in the LSTAR model and the outer (either expansion or contraction) regime in the ESTAR model.

The characteristic roots for each regime are shown in [Table 6]. Both regimes contain pairs of complex roots for both consumption and stock returns. This suggests that both variables are dynamically characterized by cyclical movements during both the expansion and contraction phases. Consumption shows symmetric movements which means in the middle regime it has explosive roots and it passes through the regime very quickly. The outer regime is rather stable, however, once it moves to the outer regime (either contraction or expansion), it stays in the outer regime longer. Stock returns have explosive roots in both regimes. Therefore they do not stay long in either regime and shows rather unstable movements.

Table 6 Characteristic roots in each regime

	Regime	Most prominent roots	Modulus
Stock Return (ESTAR)	Middle Regime (F=0)	-1.3927 0.1157 1.1345 $-0.9144 \pm 0.5520i$ $-0.6569 \pm 0.9272i$ $0.03201 \pm 0.9756i$ $0.4636 \pm 1.0483i$ $0.7831 \pm 0.3681i$	1.3927 0.1157 1.1345 1.0681 1.1363 0.9761 1.1462 0.8653
	Outer Regime (F=1)	-1.0884 $-0.8553 \pm 0.3654i$ $-0.6338 \pm 0.7431i$ $-0.1720 \pm 0.9139i$ $0.3703 \pm 0.8618i$ 0.6221 $0.6997 \pm 0.5093i$ 0.7924	1.0884 0.9300 0.9767 0.9299 0.9380 0.6221 0.8654 0.7924
Consumption Growth (ESTAR)	Middle Regime (F=0)	$-1.6031 \pm 1.1509i$ 0.4659 0.8761	1.9735 0.4659 0.8761
	Outer Regime (F=1)	$-0.6774 \pm 0.4287i$ $0.0420 \pm 0.6108i$ 0.5022	0.8016 0.6122 0.5022

V. Granger causality between consumption and stock return.

One interesting extension of consumption and stock STAR estimation is the mutual dynamic relationship using Granger causality. In particular, stock returns' effect on consumption can be re-interpreted as a test of the 'wealth effect'.

This testing, however, is carried out within a linear framework, which is not applicable in our study. Since we are using nonlinear models, we need to employ noncausality tests based on a nonlinear framework. Skalin and Terasvirta (1999) have developed such a test based on the STAR model. The authors extend the Granger noncausality test by considering an additive STAR model

$$s_t = \beta_0 + \beta_1 x_t (\theta_0 + \theta_1 x_t) F(s_{t-d}) + \delta'_1 v_t + (\delta_{20} + \delta'_2 v_t) G(y_{t-e}) + u_t \quad (9)$$

where y is the causing variable, $v_t = (y_{t-1}, \cdots, y_{t-q})'$, $\delta_j = (\delta_{j1}, \cdots, \delta_{jq})'$, $j = 1, 2$, $G(.)$ is a transition function, and e is a delay parameter. Here the noncausality hypothesis is $H_0 : \delta'_1 = \delta_{20} = \delta'_2 = 0$.

Skalin and Terasvirta (1999) point out that model (9) is not identified under the null hypothesis H_0. To overcome this identification problem, they use the following approximation Eq. (9), based on the Taylor approximation of the transition function $G(.)$;

$$s_t = \beta_0 + \beta_1 x_t (\theta_0 + \theta_1 x_t) F(s_{t-d}) + k'_1 v_t + \sum_{i=1}^{q}\sum_{j=1}^{q} \phi_{ij} y_{t-i} y_{t-j} + \sum_{i=1}^{q} \psi_i y_{t-i}^3 + u_t \quad (10)$$

where $k = (k_1, \cdots, k_q)'$, $i = 1, \cdots, q$, $j = i, \cdots, q$.

The null hypothesis that variable y does not cause variable s is $H_0 : k_i = 0$, $\phi_{ij} = 0$, $\psi = 0$. Under the null, the test statistic has an F-distribution with degrees of freedom $q(q+1)/2+2q$ in the numerator and $T - n - q(q+1)/2 - 2q$ in the denominator, where T is the number of observations and n is the dimension of the gradient vector, y_{t-i}, y_{t-i}^3, $y_{t-i}y_{t-j}$, $i = 1, \cdots, q$, $j = i, \cdots,$ q.

Nonlinear Granger causality tests are summarized in [Table 7]. According to the 'linear' results, the null hypothesis of 'stock Granger does not cause consumption' is not rejected. However, the 'nonlinear' Granger non-causality test does not accept the null hypothesis. Therefore We find that the nonlinear Granger causality test detects Granger causality from stock to consumption.

On consumption's Granger causality on stock, both linear and nonlinear tests support causality. However, the nonlinear Granger causality test shows more significant causality.

Table 7 Nonlinear (STAR) and linear Granger causation test

Nonlinear Granger Causality Test		
Caused variable	Causing variable	
	stock returns	consumption growth rate
stock returns	- - -	0.0000 0.0000
consumption growth rate	0.0005 0.0000	- - -
Linear Granger Causality Test		
Caused variable	Causing variable	
	stock returns	consumption growth rate
stock returns	- - -	0.0031 0.0019
consumption growth rate	0.1692 0.1547	- - -

Statistics are the P-values of F-test and χ-test of the null hypothesis.

VI. Conclusion

This study explores nonlinearities and dynamic properties of Korean stock returns and consumption growth rate using a STAR model. Both variables show cyclical movements and slow transition between regimes.

By extending the STAR model's empirical results, we investigate stock returns' effect on consumption or 'wealth effect' through nonlinear Granger causality tests. The linear test does not show Granger causality, however, we find significant Granger causality with a nonlinear test. This result also coincides with the previous empirical findings concerning the wealth effect of stock assets in Korea.

This research should be extended by comparing the effects before the financial crisis and after the crisis. However, we do not have enough observations after the crisis, which makes a nonlinearity test after the crisis difficult.[5)] When we have enough data sets, in the near future, hopefully we will be able to study the changes of nonlinear estimation and wealth effect.

5) Relatively long optimal lag (p*) is one of the reasons that make the nonlinear test difficult with small number of observations.

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