International Monetary and Fiscal Policy Coordination in a Liquidity Trap *

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Abstract

This paper develops an international model describing the transmission of macro shocks which push monetary policy to a 'liquidity trap, where policy is constrained by the zero lower bound on nominal interest rates, and examines the case for coordinated monetary and fiscal policy to respond to the liquidity trap. We show that financial markets tend to propagate liquidity traps, in the sense that the shocks giving rise to a liquidity trap in one country are transferred to other countries through financial markets. In a global environment economy, fiscal policy may be extremely effective in raising GDP when the economy is stuck in a liquidity trap, but it does so in a 'beggar they neighbor' fashion; when one economy is in a liquidity trap, the cross country spillover effects of fiscal policy are negative. We examine the welfare optimizing policy response to a liquidity trap when countries coordinate on monetary and fiscal policy. Our results show only a limited welfare role for using fiscal policy to exit a liquidity trap, and only a modest case for coordinated global fiscal expansion. For the most part, the country worst hit by a liquidity trap shock should use its own policies to respond, without much help from foreign policies.

Keywords:Liquidity Trap, Monetary Policy, Fiscal Policy, International Spillovers

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1 Introduction

The dramatic policy responses to the 2008-2009 global economic crisis that took place in many countries has revived discussion about the use of counter-cyclical fiscal and monetary policy. The central dilemma for policy-makers in responding to the rapid global downturn was to counter a recession precipitated by an unprecented fall in private consumption and investment spending, but at the same time having little ability to reduce nominal interest rates below their record low levels. Policy-makers followed ad hoc series of fiscal and monetary measures - government spending increases, tax cuts, and 'unconventional' monetary policy measures such as open market purchases on long-dated securities, direct increases in the monetary base, etc. In general, these policies were not obtained from the standard DSGE theoretical frameworks, but rather produced from 'back of the envelope' style arguments about the size of fiscal multipliers and the impact of liquidity injections on credit flows.

Interestingly, the issue of designing appropriate fiscal and monetary policy responses in face of a zero lower bound on nominal interest rates had been extensively debated earlier in the decade, in light of the 1990's experience of Japan. In particular, Krugman (1999), Eggertson and Woodford (2003, 2005), Jung et al. (2005), Svensson, Auerbach and Obstfeld (2004) and many other writers explored how monetary and fiscal policy could be usefully employed even when the authorities have no further room to reduce short term nominal interest rates. Recently, a number of authors have revived this literature in light of the very similar problems now encountered by the economies of Western Europe and North America. Papers by Christiano et al (2009), Eggertson (2009), Taylor et al. (2008) have explored the possibility for using government spending expansions, tax cuts, and monetary policy when the economy is in a 'liquidity trap'.

One key new aspect of the dilemma faced by authorities in the recent policy environment was that the liquidity trap was a global phenomenon. In most previous literature, analysis of the zero bound constraint on nominal interest rates focused on the problems facing either a closed economy, or a small open economy in which policy-makers in the rest of the world were not faced with the analogous constraints[1]. But when most major countries are facing similar constraints on monetary policy, it is not clear how easy or useful it is to follow the conclusions of the previous literature. First, there is a question of examining how a shock which pushes an economy beyond the zero lower bound in nominal interest rates is spread across countries. How are liquidity traps 'progagated' across countries? Secondly, policies which help to alleviate a liquidity trap in one country may exacerbate the problems in

[1] Some recent expections are Fujiwara et al. (2009), Erceg et al. (2009) and Jeanne (2009). These papers are discussed further below.

neighboring countries. The external dimension may substantially alter the effects of a given set of policy responses within a liquidity trap. Furthermore, there may be considerable benefit to coordination of policies across countries, and optimally coordinated policies may differ from the rules that would be extrapolated from the analysis of closed economies or small open economies.

This paper examines the joint determination of monetary and fiscal policies in a two country world economy where one or both countries are suspectible to negative demand shocks that precipitate a liquidity trap. The paper explores three questions relevant to the previous discussion. First, we ask how a liquidity trap is propagated across countries. In particular, taking a negative demand shock in one country which would push the unconstrained optimal nominal interest rate below zero, we ask how this is likely to impact on the policy problem in the neighboring country. Secondly, we examine the use of countercyclical fiscal policy within a liquidity trap. Recent literature has argued that fiscal policy becomes very effective when the monetary authority has no ability to adjust interest rates. We explore how this argument holds up in a global framework with separate fiscal responses. Finally, we explore the jointly optimal monetary and fiscal policy problem in an environment where fiscal and monetary authorities maximize an approximated world social welfare function. With this framework we can ask how useful is fiscal policy in responding to a liquidity trap, and how much of a contribution is made by international coordination of policy. In essence, the paper is an attempt to extend the recent literature on policy making under the zero lower bound to a global environment, where both fiscal and monetary policy are jointly determined.

The basic modeling environment we use is quite standard in the recent open economy literature. In particular, our model is essentially a convex combination of the models of Beetsma and Jensen (2005), who incorporated government spending into a two country open economy environment, and Engel (2010), who looked at an optimal policy environment with home bias in consumer preferences[2]. In both cases, the assumption is that there is a complete set of markets for consumption insurance across countries. We focus on the role of demand shocks in these models, and crucially, add the zero bound as an additional constraint on monetary policy determination.

Our central results may be summarised briefly. First, we find that with complete markets for consumption risk sharing, demand shocks which push one economy into a liquidity trap are very directly channelled across countries. In the extreme case, where preferences are identical across countries, then the liquidity trap is simultaneous and global. That is, we find

[2]Engel's paper is focused more on the role of local currency pricing for the monetary policy problem in open economies. We ignore this additional complication in our analysis - assuming the simpler framework of producer currency pricing.

that all countries enter a liquidity trap at the same time, independent of the original source of the shock. In the more realistic case with home bias in preferences, then countries may face a liquidity trap individually, depending on the strength of the shock, and the source country for the shock will first face the zero lower bound constraint on nominal interest rates. Nevertheless, the presence of complete financial markets represents a key channel for 'propagating liquidity traps' across countries.

We then go on to examine the effects of national fiscal policy responses to a liquidity trap environment. In particular, we contrast the multiplier effects of fiscal policy within a liquidity trap - either in one country or a global liquidity trap, with the equivalent responses when monetary policy is operating normally (according to a Taylor rule). The key questions are a) whether fiscal policy is more 'effective' within a liquidity trap, and b) whether fiscal expansion can generate cross country economic stimulus.

In so far as the first question is concerned, we indeed find that fiscal expansion may have a much greater effect on domestic GDP in a liquidity trap than in 'normal' times. This result relates to Christiano et al. (2009), who argue, in a closed economy setting, that the fiscal multiplier at the zero lower bound may be orders of magnitude higher than the multiplier under a Taylor rule. Interestingly, we find that in the open economy dimension, the multiplier at the zero lower bound is magnified even further. This raises an intriquing contrast with the standard intuition concerning the impacts of fiscal policy in open economies. In the standard argument, the presence of 'leakages' in an open economy will mitigate the multiplier effects of fiscal expansion in a Keynesian environment. In our analysis we find that this is true, but only if the monetary policy operates 'normally' i.e. outside a liquidity trap with the policy interest rate following a Taylor rule. In that case, a fiscal expansion in one country will cause a terms of trade appreciation, which reduces demand for home goods, and dampens the effects of the expansion on domestic GDP. By contrast, in an environment where interest rates are at the zero bound (in either the source country for the fiscal expansion or in both countries), the presence of openness will magnify the impacts of fiscal policy expansion. The key reason is that the fiscal expansion under a zero lower bound will in fact generate a large terms of trade *depreciation*, rather than appreciation. This perverse effect on the terms of trade will crowd in world spending towards the home country, and exacerbate the effects of the fiscal policy expansion, relative to that of a closed economy.

But the argument also has a flip side. With respect to question b), we find that there is no case for the large positive international spillovers of the effects of fiscal policy expansion in a liquidity trap. If monetary policy is operating under a Taylor rule, then fiscal expansion does indeed have positive international spillovers, albeit quite small, when realistically calibrated. But under a liquidity trap, the argument goes exactly the other way. We find that when a country is at the zero lower bound, the cross country spillover effects of fiscal expansion are negative, and potentially very large. So a fiscal expansion in a liquidity trap may have has a large domestic positive multiplier but by the same token, has a large international *negative* multiplier Again, the effects are related to the perverse effects on the terms of trade. A home country fiscal expansion under a liquidity trap generates a large terms of trade deterioration for the home economy, which reduces demand for foreign goods, and reduces foreign GDP. Hence, in a liquidity trap, fiscal spending is a 'beggar thy neighbor' policy.

These results would seem to undermine the case for a large coordination of expansionary fiscal policies to respond to a global liquidity trap, since fiscal expansion in one country may be very effective in reducing output gaps, but is likely to exacerbate the output gap in neighboring countries. When we examine the optimal policy problem formally, we corroborate this conjecture. We use the approximated welfare function from the underlying model to identify the optimal package of fiscal and monetary policy responses to a liquidity trap in one or both countries. Our results do show that it is always desirable to employ expansionary fiscal policy to respond to a liquidity trap. But fiscal policy will be used sparingly, and will never come close to eliminating the output gap generated by negative demand shocks. More pertinently, we find that there is relatively little international coordination of fiscal policy in an optimal policy package. The jointly optimal fiscal policy plan will require substantial fiscal expansion in the country subject to the worst of the demand shock (deeper in the liquidity trap), but relatively little expansion in the neighboring country. This is true whether this country is in a ('shallow') liquidity trap itself, or whether it is still operating with positive nominal interest rates.

We find also when one country is not constrained by a liquidity trap while the other is constrained, it is optimal for the first country to follow an expansionary monetary policy, reducing nominal interest rates to counter the negative demand shock. But it will never be optimal to to this to the extent that it fully closes its own output gap.

This paper is related to a number of recent papers that have examined policy issues in a 'zero lower bound' situation in open economies. Fujiwara et al. (2009) examine the optimal monetary problem with commitment in a multi country situation, but do not examine the determination of fiscal policy, or the transmission of demand shocks across countries. By contrast, we focus (in this draft) on the role for fiscal policy under policy discretion. Jeanne (2009) examines a 'global liquidity trap' in a model of one-period ahead pricing similar to that of Krugman (2009). Bodenstein et al (2009) use a fully specific two country DSGE model to examine the international tranmission of shocks when one country is in a liquidity trap, but do not focus on optimal monetary policy or fiscal policy choices.

The rest of the paper is organized as follows. The next section develops the basic model. Section 3 examines the efficient global equilibrium under flexible prices and endogenous fiscal policy determination. Section 4 examines the solution under sticky prices. Then in section **??** we analyze the impact of fiscal policies at the zero lower bound, and the role of international spillovers of policies. Section 5 examines the optimal policy making problem in a global cooperative agreement. Some conclusions are then offered.

2 A two country model of interacting monetary and fiscal policy

We develop a very standard two country open economy model where households consume both private and government goods, as well as supplying labour. Denote the countries as 'home' and 'foreign'. We assume that the two countries have equal population (normalized to unity), and produce a range of differentiated goods. There is a complete set of asset markets across countries, allowing for full insurance of consumption risk. Firms in each country produce private goods, while governments produce government goods and distribute them uniformly across households. Firms are not free to adjust their prices instantly however, and the presence of gradual price adjustment implies that demand shocks can have inefficient effects on output and inflation rates. We model demand shocks as country specific shocks to consumer preferences for private goods in the present relative to the future. If each monetary authority in the home and foreign country could freely adjust nominal interest rates in response to demand shocks, then an appropriate monetary policy can be designed to completely offset demand shocks. This would ensure that both countries responded to these shocks as in a frictionless first best economy. In this case, the government's fiscal policy problem would also be simply one of producing the first-best allocation between public and private goods. If however, one or both monetary authorities find that the optimal unconstrained nominal interest rate that would sustain the first best monetary policy is below zero, then monetary policy is limited by the zero lower bound on nominal interest rates. In this case, optimal fiscal and monetary policy cannot achieve the allocation of the first best frictionless economy. Then an optimal second best policy will generally make use of both fiscal and monetary policy for stabilization objectives. In this section, we first examine the impact of preference shocks in pushing one or both countries towards the zero lower bound. We then describe the effects of fiscal policy, and particulary the international transmission effects of fiscal policy, when one or both countries are constrained by the zero lower bound. In section 5, we formally derive the optimal targeting rules for monetary and fiscal policy when the zero lower bound is a constraint.

2.1 Households

As we mentioned above, the model is based on a combination of Beetsma and Jensen (2005) and Engel (2010). Let the utility of a representative infinitely lived home household evaluated from date 0 be defined as:

$$U_t = E_0 \sum_{t=0}^{\infty} (\beta)^t (U(C_t, \xi_t) - V(H_t, z_t) + J(G_t)) \tag{1}$$

where U, V, and J represent the utility of the composite home consumption bundle C_t, disutility of labour supply N_t, and utility of the government supplied public good G_t,respectively. U and J are continuously differentiable and concave in C and G, while V is differentiable and convex in N. The variable ξ_t represents a preference, or 'demand' shock, where we assume that $U_{12} > 0$, and z_t represents a 'cost shock', increasing the marginal disutility of labour supply (so that $V_{12} > 0$).

Composite consumption is defined as

$$C_t = \Phi C_{Ht}^{v/2} C_{Ft}^{1-v/2}, \quad v \geq 1$$

where $\Phi = \left(\frac{v}{2}\right)^{\frac{v}{2}} (1 - \left(\frac{v}{2}\right))^{\frac{v}{2}}$, C_{Ht} is the consumption of the home country composite good by the home household, and C_{Ft} is consumption of the foreign composite good. We allow for $v \geqslant 1$ so as to incorporate home bias in preferences[3].

In turn, C_H and C_F are defined over the range of home and foreign differentiated goods with elasticity of substitution θ between goods, so that:

$$C_H = \left[\int_0^1 C_H(i)^{1-\frac{1}{\theta}} di \right]^{\frac{1}{1-\frac{1}{\theta}}}, \qquad C_F = \left[\int_0^1 C_F(i)^{1-\frac{1}{\theta}} di \right]^{\frac{1}{1-\frac{1}{\theta}}}, \quad \theta > 1.$$

The price indices for home and foreign consumption composites are then defined as:

[3] As will be clear below, home bias is critical for the way in which the constraints leading to a liquidity trap are propagated. That home bias can have a substantial effect on optimal monetary policy in open economy models has previously been shown by Faia and Monacelli, (2008).

$$P_H = \left[\int_0^1 P_H(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \qquad P_F = \left[\int_0^1 P_F(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}},$$

while the aggregate (CPI) price index for the home country is $P = P_H^{\frac{v}{2}} P_F^{1-\frac{v}{2}}$.

Demand for individual differentiated home and foreign goods and home and foreign composite goods may be obtained from these functions in the usual way. Each firm faces a demand elasticity of θ.

It is assumed that home government spending falls exclusively on the home composite good, and analogously for the foreign composite good. In their choice of individual consumption and labour supply, households and firms takes the total consumption of government spending as given.

The representative home household sells labour services to each of a continuum of home country firms, receiving an average nominal wage W_t in return. The household's implicit labour supply is determined by the condition:

$$U_C(C_t, \xi_t) W_t = P_t V'(H_t, z_t). \tag{2}$$

The assumption of a complete set of state contingent international assets markets implies that state-contingent nominal marginal utility is equated across countries. This simplies a relationship between the ratio of marginal utilities and the real exchange rate as follows:

$$U_C(C_t, \xi_t) = U_C(C_t^*, \xi_t^*) \frac{S_t P_t^*}{P_t} = U_C(C_t^*, \xi_t^*) T_t^{v-1}, \tag{3}$$

where S_t is the nominal exchange rate (home price of foreign currency), $P_t^* = P_F^{*v/2} P_H^{*1-v/2}$ is the foreign CPI, and $T = \frac{SP_F^*}{P_H}$ is the home country terms of trade. Implicit in this condition is the assumption that the law of one price holds in individual goods and home and foreign composite consumption goods (i.e. so that $P_F = SP_F^*$, etc).

In addition, we assume that households have access to a market in domestic nominal government bonds, each of which pays an interest rate of R_t in all states of the world. Thus we can define an Euler equation for nominal bond pricing given by:

$$\frac{U_C(C_t, \xi_t)}{P_t} = E_t \beta \frac{U_C(C_{t+1}, \xi_{t+1})}{P_{t+1}} R_{t+1}. \tag{4}$$

Foreign households preferences and choices can be defined exactly symmetrically. The foreign representative household has weight $v/2$, $(1-v/2)$ on the foreign (home) compositive good in preferences.

2.2 Firms

Firms employ labour to produce individual differentiated goods. Firm i in the home economy uses the production function:

$$Y_t(i) = AN_t(i),$$

where A is a common economy-wide productivity term that applies to all home firms. The home firm has period t profits defined by $\Pi_t(i) = P_{Ht}(i)Y_t(i) - W_t H_t(i)(1 - s_t)$, where s_t is a wage subsidy offered to each home firm by the home government, financed with lump-sum taxation. In line with much previous literature, we assume that each home firm re-sets its price according to Calvo pricing, where the probability of re-adjusting its price is $1 - \kappa$ in each period.

In selling to home and foreign consumers, as well as the home government, the home firm faces a constant elasticity of demand θ. Thus, when it can adjust its price the home firm sets the price that maximizes the present value of profits evaluated using the stochastic discount factor $m_{t+j} = \frac{P_t}{U_C(C_t,\varepsilon_t)} \frac{U_C(C_{t+j},\xi_{t+j})}{P_{t+j}}$. This leads to the optimal price setting condition as follows:

$$\widetilde{P}_{Ht}(i) = \frac{\theta}{\theta - 1}(1 - s_t) \frac{E_t \sum_{j=0} m_{t+j}\kappa^j \frac{W_{t+j}}{A_{t+j}} Y_{t+j}(i)}{E_t \sum_{j=0} m_{t+j}\kappa^j Y_{t+j}(i)}. \tag{5}$$

Facing a common marginal cost, all home firms that can adjust their price choose the same price. In the aggregate, the price index for the home good then follows the process given by:

$$P_{Ht} = [(1 - \kappa)\widetilde{P}_{Ht}^{1-\theta} + \kappa P_{Ht-1}^{1-\theta}]^{\frac{1}{1-\theta}}. \tag{6}$$

The behaviour of foreign firms and the foreign good price index may be described analogously.

2.3 Government and Fiscal Policy Choices

We assume that governments have access to lump sum taxation. Each government then has the task of choosing both an optimal subsidy to its domestic monopoly producers and the level of public goods spending for its domestic constituents. It is easy to see that the optimal subsidy will offset the monopoly markup exactly, so that $s = 1/\theta$ will obtain in each country for all periods. As regards the determination of public spending, in the analysis below, we will focus on a jointly optimal cooperative monetary and fiscal policy set to maximize the sum of home and foreign utility. In this regard the determination of government spending will be set as a trade-off between alternative objectives, and will be depend critically on the constraints on monetary policy. If monetary policy is unrestricted, so that central banks in each country can freely adjust nominal interest rates, then (as shown below) the global equilibrium supports a first best flexible price allocation, and government spending is set optimally from a pure public finance perspective. In a liquidity trap however, government spending policies will typically deviate from the optimal public finance rule and will be chosen to satisfy stabilization objectives.

The optimal public spending rule under an efficient flexible price allocation is defined in section 3 below. The determination of optimal government spending in a liquidity trap will be the subject of section 5 below.

2.4 Monetary Policy

As mentioned, monetary policy may or may not be constrained by the zero lower bound on nominal interest rates. If it is unconstrained, then there is a range of monetary policy instrument rules that can implement an optimal targeting rule as defined below. For comparison purposes, however, it is sometimes useful to be more specific in the definition of monetary rules. Therefore, in some of the discussion below, we will make use of the explicit assumption that monetary policy follows a modified Taylor rule, given by:

$$R_t^r = (1+\rho_t)(1+\widetilde{\pi})\left(\frac{P_t}{P_{t-1}}\frac{1}{1+\widetilde{\pi}}\right)^{\gamma} \tag{7}$$

where ρ_t represents a desired path for the equilibrium real interest rate, and $\widetilde{\pi}$ represents a desired path for the inflation rate. We assume that $\gamma > 1$. This rule is somewhat unrealistic in that we do not allow for interest rate 'smoothing'. This is not critical for the results, however.

The monetary authority can follow the rule (7) only when $R_t^r > 1$ however. If the rule stipulates a negative nominal interest rate, then the central bank is constrained by the zero lower bound on nominal interest rates. Thus, the path of nominal interest rates in the model must be governed by:

$$R_t = \max\left(R_t^r, 1\right). \tag{8}$$

Again, the monetary authority of the foreign country is characterized in an analogous manner.

2.5 Market Clearing

Each home country firm i faces demand for its good from home consumers, foreign consumers and its home government. Using the properties of demand curves from above, we can define equilibrium in the market for good i as

$$Y_{Ht}(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta}\left[\frac{v}{2}\frac{P_{Ht}}{P_t}C_t + (1-\frac{v}{2})\frac{P_{Ht}}{S_tP_t^*}C_t^* + G_t\right],$$

where G_t represents total home government spending on the aggregate home good. Now, aggregating across all home firms, market clearing in the home good is defined as

$$Y_{Ht} = \frac{v}{2}\frac{P_{Ht}}{P_t}C_t + (1-\frac{v}{2})\frac{P_{Ht}}{S_tP_t^*}C_t^* + G_t. \tag{9}$$

Here $Y_{Ht} = V_t^{-1}\int_0^1 Y_{Ht}(i)di$ is aggregate home country output, where we have defined $V_t = \int_0^1 \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} di$. It follows that home country employment (employment for the representative individual home household) is given by $N_t = \int_0^1 N(i)di = A^{-1}Y_{Ht}V_t$.

In the same manner, we may write the aggregate market clearing condition for the foreign good as

$$Y_{Ft} = \frac{v}{2}\frac{P_{Ft}^*}{P_t^*}C_t^* + (1-\frac{v}{2})\frac{S_tP_{Ft}^*}{P_t}C_t^* + G_t^*, \tag{10}$$

and again, we may define foreign employment as: $N_t^* = \int_0^1 N_t^*(i)di = A^{*-1}Y_{Ft}V_t^*$, where $V_t^* = \int_0^1 \left(\frac{P_{Ft}^*(i)}{P_{Ft}^*}\right)^{-\theta} di$.

An equilibrium in the world economy with positive nominal interest rates may be described by the equations (3), and (2), (4), (5) (6) and (7) for the home and foreign economy, as well as (9) and (10). For given values of V_t and V_t^*, and given government spending policies, these equations determine an equilibrium sequence for the variables $C_t, C_t^*, W_t, W_t^*, S_t, P_{Ht}$, $P_{Ft}^*, \widetilde{P}_{Ht}, \widetilde{P}_{Ft}^*$, R_t, R_t^*, and N_t, N_t^*. In a log-linear approximation of the model, the distribution expressions V_t and V_t^* drop out, so up to a first order approximation, the behavior of all variables is fully determined by the outlined equations. When one or both countries is constrained by the zero lower bound, we have to define equilibrium in a constrained manner, as described below.

3 Flexible Price Equilibrium and Optimal Monetary and Fiscal Policy

We first define the equilibrium of the model in a fully flexible price world equilibrium, where $\kappa = 0$ in each country. In that case, $P_{Ht}(i) = P_{Ht}$, $P_{Ft}(i) = P_{Ft}$, and $V_t = V_t^* = 1$. In addition (given the presence of optimal subsidies) we have $P_{Ht} = A^{-1}W_t$ and $P_{Ft}^* = A^{-1}W_t^*$. Define $T_t = \frac{SP_{Ft}}{P_{Ht}}$ as the home country terms of trade. Then we may describe a flexible price efficient equilibrium by the equations:

$$U_C(\widetilde{C}_t^*, \xi_t)A = \widetilde{T}_t^{1-v/2} V'(\widetilde{N}_t, \zeta_t), \qquad U_C^*(\widetilde{C}_t^*, \xi_t^*)A^* = \widetilde{T}_t^{1-v/2} V'(\widetilde{H}_t^*, \zeta_t^*) \tag{11}$$

$$U_C(\widetilde{C}_t^*, \xi_t) = U_C^*(\widetilde{C}_t^*, \xi_t^*) T_t^{v-1}, \tag{12}$$

$$A\widetilde{N}_t = \frac{v}{2} \widetilde{T}_t^{1-v/2} \widetilde{C}_t + (1 - \frac{v}{2}) \widetilde{T}_t^{v/2} \widetilde{C}_t^* + \widetilde{G}_t, \tag{13}$$

$$A^* \widetilde{N}_t^* = \frac{v}{2} \widetilde{T}_t^{-(1-v/2)} \widetilde{C}_t^* + (1 - \frac{v}{2}) \widetilde{T}_t^{-v/2} \widetilde{C}_t + \widetilde{G}_t^* \tag{14}$$

This implicitly describes the efficient world equilibrium for consumption, output (or employment), and the terms of trade, conditional on values for home and foreign rates of government spending. But government spending policies are endogenously chosen by governments in each country to maximize a world social welfare function. In the Appendix, it is shown that, under fully flexible prices, optimal government spending policies in each country are characterized by the conditions:

$$J_G(\widetilde{G}_t)A = V'(\widetilde{N}_t, \zeta_t), \qquad J_G^*(\widetilde{G}_t^*)A^* = V'(\widetilde{N}_t^*, \zeta_t^*). \tag{15}$$

Since government spending falls exclusively on home produced goods, the optimal spending rates trade off the marginal disutility of hours worked against the marginal utility of hours spent producing public goods.

Although monetary policy is entirely neutral in this setting, we can also define the nominal interest rate that is associated with an optimal global flexible price equilibrium. We assume that an optimal policy is to set inflation to zero. There is no generality lost in making this assumption, since as we see below, an optimal unconstrained monetary policy in an environment of sticky prices will also set domestic inflation rates to zero. Hence, the nominal interest rates that supports the optimal global flexible price equilibrium for the home and foreign countries are given by[4]:

$$\widetilde{R}_{t+1} = \frac{1}{\beta}\frac{U_C(C_t,\xi_t)}{P_t E_t \frac{U_C(C_{t+1},\xi_{t+1})}{P_{t+1}}},\ \widetilde{R}^*_{t+1} = \frac{1}{\beta}\frac{U^*_C(C^*_t,\xi^*_t)}{P^*_t E_t \frac{U^*_C(C^*_{t+1},\xi^*_{t+1})}{P^*_{t+1}}} \tag{16}$$

In an economy with sticky prices as described below, if monetary authorities can set interest rates as in (16) without violating the zero lower bound constraint, then a globally efficient allocation can be supported. Thus, these conditions give us the key information as to when each country's optimal monetary policy will be constrained by the zero bound on nominal interest rates.

It is instructive to further analyze equations (11)-(16) by taking a linear approximation around the globally efficient steady state. In what follows, we restrict ourselves to the analysis of preference (or 'demand') shocks ξ_t and ξ^*_t. While productivity shocks ζ_t and ζ^*_t can in principle push the economy towards the lower bound of nominal interest rates, they do so in a manner that simultaneously increases total produced output. It is more realistic and intuitive to assume that the zero lower bound is approached following a collapse in demand in one or both countries (by a fall in ξ_t or ξ^*_t).

For a given variable X, define $\widetilde{x} = \ln(\widetilde{X}/\overline{X})$ to be the log difference of the global efficient value from the steady state, except for $\varepsilon_t \equiv \ln(\xi_t)$, and $\widetilde{\pi}_{Ht+1}$ and $\widetilde{r}_t$, which refer respectively to the *level* of the inflation rate and nominal interest rate. Normalize also so that $A = 1$, and thus $T = 1$ in a steady state. Then we may express the linear approximation of (11)-(16) as:

$$\sigma\widetilde{c}_t - \varepsilon_t + \phi\widetilde{n}_t + (1-\frac{v}{2})\widetilde{\tau}_t = 0 \tag{17}$$

$$\sigma\widetilde{c}^*_t - \varepsilon^*_t + \phi\widetilde{n}^*_t - (1-\frac{v}{2})\widetilde{\tau}_t = 0 \tag{18}$$

$$\widetilde{n}_t = cy(\frac{v}{2}\widetilde{c}_t + (1-\frac{v}{2})\widetilde{c}^*_t) + 2\frac{v}{2}(1-\frac{v}{2})\widetilde{\tau}_t + (1-cy)\widetilde{g}_t, \tag{19}$$

$$\widetilde{n}^*_t = cy(\frac{v}{2}\widetilde{c}^*_t + (1-\frac{v}{2})\widetilde{c}_t) - 2\frac{v}{2}(1-\frac{v}{2})\widetilde{\tau}_t + (1-cy)\widetilde{g}^*_t, \tag{20}$$

$$\sigma\widetilde{c}_t - \varepsilon_t = \sigma\widetilde{c}^*_t - \varepsilon^*_t + (v-1)\widetilde{\tau}_t, \tag{21}$$

[4]These represent *equilibrium* values for nominal interest rates, when inflation rates are equal to zero. In order to implement this outcome, the monetary authorities must follow a rule in which the price levels in each country are determinate. The Taylor rule given by (7) where $\gamma > 1$ is an example of such a rule. In addition, for this monetary policy to be feasible in the flexible price equilibrium, the nominal interest rates must satisfy condition (8) (and the equivalent condition for the foreign economy). Thus, the zero lower bound constraint must be satisfied even in a flexible price economy. We do not pursue the implications of the zero lower bound in this setting, because in a purely flexible price economy it would be easy for the monetary authority to satisfy the zero lower bound at any value of the real interest rate simply by letting the inflation rate adjust costlessly.

$$\sigma_g \widetilde{g}_t + \phi \widetilde{n}_t = 0, \tag{22}$$

$$\sigma_g \widetilde{g}_t + \phi \widetilde{n}_t^* = 0. \tag{23}$$

Here we have defined $\sigma \equiv -\frac{U_{CC}\overline{C}}{U_C}$ as the inverse of the elasticity of intertemporal substitution in consumption, $\phi \equiv -\frac{V''\overline{H}}{V'}$ as the elasticity of the marginal disutility of hours worked and $\sigma_g \equiv -\frac{J''\overline{G}}{J'}$ as the elasticity of marginal utility of public goods. Finally, $\varepsilon_t = \frac{U_{C\xi}}{U_C} \ln(\xi_t)$ is the measure of a positive demand shock in the home country, with an equivalent definition for the foreign country.

We may solve the system (17)-(23) to obtain the first order solutions for consumption, output and the terms of trade in response to demand shocks. For any variable x, define $x^W = \frac{x+x^*}{2}$ as the world average, and $x^R = \frac{x-x^*}{2}$ as the world relative in the variable. Since then $x = x^W + x^R$, we can write home and foreign consumption responses to demand shocks as:

$$\widetilde{c}_t = \frac{1}{\sigma}(1 - \frac{\phi c_y}{\phi + \sigma})\varepsilon_t^W + \frac{1}{\sigma}(1 - \frac{\phi c_y (1-v)^2}{\Delta})\varepsilon_t^R$$

$$\widetilde{c}_t^* = \frac{1}{\sigma}(1 - \frac{\phi c_y}{\phi + \sigma})\varepsilon_t^W - \frac{1}{\sigma}(1 - \frac{\phi c_y (1-v)^2}{\Delta})\varepsilon_t^R$$

where $\Delta = \phi c_y(\sigma v(2-v) + (1-v)^2) + \phi(1-c_y) + \sigma > 0$.

A demand shock raises the efficient flexible price world consumption level, but the impact on individual country consumption depends on the source of the demand shock. A world demand shock will raise the flexible price level of home and foreign consumption equally, but a relative home country demand shock will raise the flexible price home consumption, while reducing the flexible price level of foreign consumption.

The impact of demand shocks on flexible price output levels are likewise written as:

$$\widetilde{n}_t = \frac{c_y}{\phi + \sigma}\varepsilon_t^W + \frac{c_y(v-1)}{\Delta}\varepsilon_t^R$$

$$\widetilde{n}_t^* = \frac{c_y}{\phi + \sigma}\varepsilon_t^W - \frac{c_y(v-1)}{\Delta}\varepsilon_t^R$$

A world demand shock raises equilibrium output in both countries. When there is home bias in preferences, so that $v > 1$, a relative home demand shock tends to raise home output and reduce foreign output.

Demand shocks also affect the flexible price efficient response of the terms of trade. We can show that:

$$\widetilde{\tau}_t = -\frac{\phi c_y (v-1)}{\Delta} \varepsilon_t^R$$

A home relative demand shock will cause a terms of trade improvement, raising the relative price of the home good.

If monetary authorities can adjust nominal interest rates freely to respond to demand shocks, then the flexible price efficient global equilibrium can be sustained. Therefore, the critical issue as regards the limits of monetary policy is the impact of the demand shock on the flexible price efficient (or 'Wicksellian') nominal interest rate. Denote $\widetilde{r}_t$ as the (level of the) net nominal interest rate, and $\overline{r}$ its steady state value. The a log linear approximation of (16) leads to the expressions for the flexible price equilibrium nominal interest rate in the home country as:

$$\widetilde{r}_t = \overline{r} + \sigma E_t(\widetilde{c}_{t+1} - \widetilde{c}_t) - E_t(\varepsilon_{t+1} - \varepsilon_t) + E_t \widetilde{\pi}_{Ht+1} + (1 - \frac{v}{2}) E_t(\widetilde{\tau}_{t+1} - \widetilde{\tau}_t) \qquad (24)$$

Assume that an efficient monetary policy is to keep the domestic rate of inflation equal to zero. In addition, for now, assume that demand shocks follow and AR(1) process so that $\varepsilon_{t+1} = \mu \varepsilon_t + u_t$ and $\varepsilon^*_{t+1} = \mu \varepsilon^*_t + u^*_t$, where u_t and u^*_t are mean-zero and i.i.d., then the value of $\widetilde{r}_t$ when the right hand side is driven by demand shocks alone can be derived as:

$$\widetilde{r}_t = \overline{r} + \left(\frac{\phi c_y}{\phi + \sigma} \varepsilon_t^W + \frac{\phi c_y (v-1)}{\Delta} \varepsilon_t^R \right) (1 - \mu)$$

In similar manner, the foreign efficient nominal interest rate is:

$$\widetilde{r}^*_t = \overline{r} + \left(\frac{\phi c_y}{\phi + \sigma} \varepsilon_t^W - \frac{\phi c_y (v-1)}{\Delta} \varepsilon_t^R \right) (1 - \mu)$$

A negative global demand shock reduces the efficient interest rate in both countries. The impact of a country specific demand shock depends on the degree of home bias in preferences. When preferences are identical, without home bias, then the efficient interest rate is equalized across countries. With a positive degree of home bias however, a negative home demand shock will reduce the efficient home interest rate more than the foreign interest rate. In this limit, as $v \to 2$, then interest rates are affected only by own-country demand shocks, since in this case the economies are effectively closed.

This analysis has direct bearing on the occurance of a liquidity trap in the global economy, precipitated by demand shocks. In the case of identical home and foreign preferences, interest rates are equalized. In this case, if a negative demand shock is large enough such that $\widetilde{r}_t < 0$, then both countries will fall into a liquidity trap simultaneously, irrespective of the origination of the demand shock. On the other hand, when $v > 1$, each country's efficient interest rate is more sensitive to domestic demand shocks. Thus, for a negative home country demand shock, the home country will hit the liquidity trap first, since $\widetilde{r}_t - \widetilde{r}_t^* < 0$. Thus, whether one or both countries fall into a liquidity trap in this case depends on the degree of home bias, and the size of the shock.

4 Monetary and Fiscal Policy with Sticky Prices

When prices are sticky, we may derive a log-linear approximation of the model in terms of inflation and output gaps in a similar manner to Clarida et al. (2002) and Engel (2010). Let $x = \ln(X_t/\overline{X})$ be the log deviation of any variable from its steady state (again, inflation and nominal interest rates are in levels). Taking a linear approximation of (2), (5) and (6) around the zero inflation steady state, we derive the standard forward looking inflation equation, given by:

$$\pi_{Ht} = k(\phi n_t + \sigma c_t - \varepsilon_t + (1 - \frac{v}{2})\tau_t) + \beta E_t \pi_{Ht+1} \tag{25}$$

where $k = \frac{(1-\beta\kappa)(1-\kappa)}{\kappa}$.

Using (3), (9) and (10), we can solve for c_t and τ_t in terms of n_t, n_t^*, ε_t and ε_t^*. This gives

$$c_t = \frac{v(2-v)}{D}\varepsilon_t^R + \frac{(n_t - (1-c_y)g_t)(D+v-1)}{2c_y D} + \frac{(n_t^* - (1-c_y)g_t^*)(D-(v-1))}{2c_y D} \tag{26}$$

$$c_t^* = -\frac{v(2-v)}{D}\varepsilon_t^R + \frac{(n_t - (1-c_y)g_t)(D+v-1)}{2c_y D} + \frac{(n_t^* - (1-c_y)g_t^*)(D-(v-1))}{2c_y D} \tag{27}$$

$$\tau_t = -\frac{v(v-1)}{D}\varepsilon_t^R + \frac{\sigma\left((n_t - (1-c_y)g_t) - (n_t^* - (1-c_y)g_t^*)\right)}{c_y D} \tag{28}$$

Here, we define $D = \sigma v(2-v) + (1-v)^2$. Then, using (26) -(28), and using the fact that in a flexible price efficient equilibrium, $\phi\widetilde{n}_t + \sigma\widetilde{c}_t - \varepsilon_t + (1 - \frac{v}{2})\widetilde{\tau}_t = 0$, we may re-write (25) in terms of 'gaps', where for a variable x, $\widehat{x} = x - \widetilde{x}$. Thus, we have

$$\pi_{Ht} = k(\phi\widehat{n}_t + \frac{\sigma}{2c_y D}\left[\widehat{ng}_t(1+D) + \widehat{ng}_t^*(D-1)\right]) + \beta E_t \pi_{Ht+1} \tag{29}$$

where $\widehat{ng}_t = (\widehat{n}_t - (1-c_y)\widehat{g}_t)$ and $\widehat{ng}_t^* = (\widehat{n}_t^* - (1-c_y)\widehat{g}_t^*)$. Thus, domestic inflation is driven by both domestic and foreign output gaps, and domestic and foreign government spending gaps. In the special case where $\sigma = 1$, $D = 1$, and we recover Clarida et al's (2002) result that the inflation equation is independent of the foreign output gap (and also the foreign government spending gap).

In a similar manner, we can derive the 'dynamic IS equation' in terms of gaps, using (26)-(28) and the log linearized Euler equation (4). This gives

$$\begin{aligned} & E_t(\widehat{ng}_{t+1} - \widehat{ng}_t)(D + v - 1) + E_t(\widehat{ng}^*_{t+1} - \widehat{ng}^*_t)(D - (v - 1)) \\ = \ & \frac{2c_y D}{\sigma} E_t \left(r_t - \widetilde{r}_t - \pi_{Ht+1} - (1 - \frac{v}{2}) \frac{\sigma \left((\widehat{ng}_{t+1} - \widehat{ng}^*_{t+1}) - (\widehat{ng}_t - \widehat{ng}^*_t) \right)}{c_y D} \right) \end{aligned} \tag{30}$$

An identical set of derivations given the equations for the foreign economy as:

$$\pi^*_{Ft} = k(\phi \widehat{n}^*_t + \frac{\sigma}{2c_y D} \left[\widehat{ng}^*_t(1 + D) + \widehat{ng}_t(D - 1) \right]) + \beta E_t \pi^*_{Ft+1} \tag{31}$$

$$\begin{aligned} & E_t(\widehat{ng}^*_{t+1} - \widehat{ng}^*_t)(D + v - 1) + E_t(\widehat{ng}_{t+1} - \widehat{ng}_t)(D - (v - 1)) \\ = \ & \frac{2c_y D}{\sigma} E_t \left(r^*_t - \widetilde{r}^*_t - \pi_{Ft+1} + (1 - \frac{v}{2}) \frac{\sigma \left((\widehat{ng}_{t+1} - \widehat{ng}^*_{t+1}) - (\widehat{ng}_t - \widehat{ng}^*_t) \right)}{c_y D} \right) \end{aligned} \tag{32}$$

The system (29)-(32) may be solved for the four variables $\widehat{n}_t$, $\widehat{n}^*_t$, $\widehat{\pi}_{Ht}$, and $\widehat{\pi}^*_{Ft}$, given an assumption about the determination of the policy interest rates r_t and r^*_t in each country. First we note as stated above that if policy interest rates can be adjusted without constraint, then all 'gaps' may be closed from(29)-(32) using monetary and fiscal policy in conjunction. In this case, we have $r_t = \widetilde{r}_t$, $r^*_t = \widetilde{r}^*_t$, and $\widehat{n}_t = \widehat{n}^*_t = \widehat{g}_t = \widehat{g}^*_t = \pi_{Ht} = \pi_{Ft} = 0$. We focus however on cases where either $\widetilde{r}_t$, $\widetilde{r}^*_t$, or both are below zero, so the policy interest rate cannot achieve the Wicksellian equilibrium interest rate and simultaneously achieve zero inflation.

4.1 The use of fiscal policy in a liquidity trap

We wish to examine the international transmission of fiscal policy when one or both countries is in a liquidity trap. In order to do this, it is useful to contrast the impact of fiscal policy in a liquidity trap to that when the economy is operating under a positive nominal interest rate and following a Taylor rule according to (7). While it is true that an optimal combination of monetary and fiscal policy can eliminate all welfare gaps in the case where nominal interest rates are positive, it is still instructive to contrast the impact of shocks when the economy is above the zero lower bound (so that nominal interest rates can be adjusted freely) with that when the economy is stuck in a liquidity trap. Hence, in this section, we specialize the Taylor rule (7) to the case where the desired real real interest rate $\overline{r}$ is constant (i.e. assuming that the monetary and fiscal authorities do not follow a policy of closing all gaps in the economy with positive nominal interest rates). We also assume that the target inflation rate $\widehat{\pi}$ is zero. Thus, in the linearized versions of (7) we set $r_t = \overline{r} + \gamma\pi_{Ht}$, and $r_t^* = \overline{r} + \gamma\pi_{Ft}^*$.

It is first helpful to focus on the special case where $v = 1$, so there is no home bias and preferences are identical across countries. In this case, using the results of section () above, countries have identical Wicksellian interest rates, and hence, when negative demand shocks are large enough, both are constrained by the zero lower bound simultaneously. From (16) then we can work out the impact of savings shocks which push the economy into the zero lower bound.

First, focus on the case where interest rates are above the zero lower bound. In addition, assume that the coefficients on inflation in the interest rate rules are identical across countries. We assume that a shock to the Wicksellian interest rate $\widetilde{r}_t$ has persistence $0 < \mu < 1$, so that $\widetilde{r}_{t+1} = \mu\widetilde{r}_t + v_{t+1}$, where v_t is i.i.d. and mean zero. Then, we may derive the impact of these shocks on home and foreign output as follows:

$$\widehat{n}_{t(\text{Taylor})} = \widehat{n}^*_{t(\text{Taylor})} = \frac{(1-\beta\mu)\widetilde{r}_t}{\Delta_1} \tag{33}$$

where $\Delta_1 = \sigma(1-\beta\mu)(1-\mu) + k(\sigma+\phi c_y)(\gamma-\mu) > 0$. Thus, a negative preference shock reduces the output gap in both countries equally. But fiscal spending shocks have differential effects. Assume that shocks to the fiscal gaps $\widehat{g}_t$ and $\widehat{g}_t^*$ are also i.i.d. with persistence μ.

$$\begin{aligned}\widehat{n}^g_{t(\text{Taylor})} &= \frac{1}{2}(1-c_y)\frac{[2\Delta_1 + \kappa\phi c_y(\sigma-1)(\gamma-\mu)]\left((1-\beta\mu)(1-\mu)+\kappa(\gamma-\mu)\right)\widehat{g}_t}{\Delta_1\Delta_2} \\ &\quad + \frac{1}{2}(1-c_y)\frac{[\kappa\phi c_y(\sigma-1)(\gamma-\mu)]\left((1-\beta\mu)(1-\mu)+\kappa(\gamma-\mu)\right)\widehat{g}_t^*}{\Delta_1\Delta_2}\end{aligned} \tag{34}$$

where $\Delta_2 = (1-\beta\mu)(1-\mu) - k(1+\phi c_y)(\gamma-\mu) > 0$. When nominal interest rates are positive, and the Taylor rule applies ($\gamma > 1$), then shocks to the fiscal spending gap in either the home country or the foreign country will raise the home output gap (here and in what follows, we make the assumption that $\sigma > 1$). Thus, the international transmission of fiscal policy is positive.

The response of the terms of trade can be derived as follows:

$$\widehat{\tau}^g_{t(\text{Taylor})} = -\frac{\phi k(1-c_y)(\gamma-\mu)(\widehat{g}_t - \widehat{g}_t^*)}{\Delta_2} \tag{35}$$

Under a Taylor rule, a domestic fiscal expansion causes a terms of trade appreciation. The impact on home country consumption is given by:

$$\widetilde{c}^{g}_{t(\text{Taylor})} = -\frac{\phi k(1-c_y)(\gamma-\mu)(\widehat{g}_t+\widehat{g}^*_t)}{2\Delta_1} \tag{36}$$

This is always negative. Since this is the case where $v = 1$, home and foreign consumption fall equally in response to a fiscal gap expansion in either country.

A domestic fiscal expansion under a positive interest rate rule leads to a direct expansion of aggregate demand and domestic output. This raises domestic marginal cost, increasing inflation. The rise in inflation leads to a rise in the domestic nominal interest rate. Since real interest rates are equated in this case, there must be an anticipated terms of trade depreciation for the home economy so as to satisfy interest rate parity. Given that the fiscal expansion is temporary, this implies an immediate terms of trade appreciation . The effect on foreign output is then twofold. First, the fiscal expansion leads to a fall in aggregate consumption (which is equalized across countries in the case $v = 1$) and a fall in demand for foreign output. Secondly, the foreign terms of trade depreciation leads to a rise in demand for foreign output. For $\sigma > 1$, the first effect dominates, and foreign output increases.

How large are the fiscal multipliers in the economy with a Taylor rule? It is easy to show that both the own country and cross country fiscal multipliers must be less than unity. We define the fiscal spending multiplier as $\frac{dY}{dG} = \frac{Y}{G}\frac{\widehat{n}}{\widehat{g}} = \frac{1}{1-cy}\frac{\widehat{n}}{\widehat{g}}$. From (35) and (36), we see that since both consumption and the terms of trade fall following a fiscal spending shock, it must be the case that output rises by less that $(1-c_y)\widehat{g}_t$. It follows immediately that the government spending multiplier in the economy governed by a Taylor rule is less that unity.

4.2 Demand shocks and Fiscal Policy at the Zero Bound

Now, let us look at the same case, but assuming that the zero lower bound binds. As we noted, the zero bound hits both countries at the same time, since the Wicksellian interest rate is identical across countries. Note that a demand shocks which reduces the Wicksellian interest rate is identical to an exogenous increase in desired savings, so we refer to this as a 'savings shock'. We wish to describe the impact of savings shock on each country's output, as well as derive the domestic and international effects of government spending shocks at the zero lower bound. In order to draw out a clear comparison to the previous case where the Taylor rule applied, we make the following assumption. Assume that at time t there is an unanticipated fall in $\widetilde{r}_t$ to $\widetilde{r}_L < 0$. Assume that $\widetilde{r}_t$ reverts back to $\overline{r}$ with probability $1-\mu$ in each period henceforth, and then remains at $\overline{r}$ thereafter. In addition, we make the parallel assumption about the fiscal policy shocks. So long as the $\widetilde{r}_t = \widetilde{r}_L$, the fiscal gaps are non-zero. Once the interest rate is above zero, all fiscal gaps revert back to zero. While this is somewhat of an artificial experiment, it has two advantages in terms of exposition. First, it allows for a direct comparison to the expressions for the impact of persistent savings and fiscal shocks in the economy operating under a Taylor rule. Second, it corresponds to the structure of optimal discretionary monetary and fiscal policy explored in the next section.

Following these assumptions, the impact of savings shocks on domestic and foreign output can be derived as

$$\widehat{n}_t^r = \widehat{n}_t^{*r} = \frac{(1-\beta\mu)\widetilde{r}_t}{\Delta_3} \tag{37}$$

where $\Delta_3 = \sigma(1-\beta\mu)(1-\mu) - \mu k(\sigma + \phi c_y)$. Note again, due to the assumption of no home bias, the impact of the savings shock on home and foreign output is identical when both countries are constrained to zero nominal interest rates.

In order that the equilibrium be determinate, it is necessary that $\Delta_3 > 0$. This puts a limit on the degree of persistence of the savings shock that can be accomodated under this analysis. We make this assumption in what follows.

Comparing (33) and (37), we can establish that

$$\widehat{n}_{t(\mathrm{ZLB})}^r - \widehat{n}_{t(\mathrm{Taylor})}^r = \frac{\gamma\kappa c_y(\sigma + \phi c_y)(1-\mu)(1-\beta\mu)\widetilde{r}_t}{\Delta_1\Delta_3}$$

For a negative savings shock, output falls more when the economy is constrained by the zero lower bound than when interest rates can adjust. This is a familiar result from Eggertson and Woodford (2003) and Eichenbaum et al. (2009). Under a Taylor rule the fall in aggregate demand leads to a fall in inflation and a compensating fall in the nominal and ex-post real interest rate in each country. When both countries are constrained by the zero bound, the fall in expected inflation leads to a *rise* in the real interest rate, leading to a further fall in aggregate demand. So long as $\Delta_3 > 0$, this process has an equilibrium outcome where there is a large fall in output in both countries, which reduces savings sufficiently to adjust to the higher real interest rates.

The impact of shocks to the fiscal gaps on home country output at the zero lower bound can be derived as:

$$\begin{aligned} \widehat{n}_{t(\mathrm{ZLB})}^g \quad = \quad & \frac{1}{2}(1-c_y)\frac{[2\Delta_3 - \mu\kappa\phi c_y(\sigma-1)]\left((1-\beta\mu)(1-\mu)+\mu\kappa\right)\widehat{g}_t}{\Delta_3\Delta_4} \\ & + \frac{1}{2}(1-c_y)\frac{-\left[\mu\kappa\phi c_y(\sigma-1)\right]\left((1-\beta\mu)(1-\mu)-\mu\kappa\right)\widehat{g}_t^*}{\Delta_3\Delta_4} \end{aligned} \tag{38}$$

where $\Delta_4 = (1-\beta\mu)(1-\mu) - \mu k(1+\phi c_y) > 0$,which again must be positive for determinacy.

We will compare the multiplier effects of domestic fiscal expansion in (38) with the equivalent under the Taylor rule below. Here, we note that from (38), the impact of foreign fiscal expansion is *negative.* That is, under a global liquidity trap, fiscal expansion has a *beggar thy neighbor* characteristic, reducing the output of foreign countries.

The derivation of the terms of trade and consumption in the liquidity trap can be expressed simply. The terms of trade responds as:

$$\widehat{\tau}^{g}_{t(\text{ZLB})} = \frac{\mu\phi k(1-c_y)(\widehat{g}_t - \widehat{g}^{*}_t)}{\Delta_4}$$

Under a Taylor rule, a domestic fiscal expansion causes a terms of trade depreciation. The impact on home country consumption is given by:

$$\widehat{c}^{g}_{t(\text{ZLB})} = \frac{\mu\phi k(1-c_y)(\widehat{g}_t + \widehat{g}^{*}_t)}{2\Delta_3}$$

In the zero lower bound, a home country fiscal expansion generates a terms of trade depreciation, and a rise in home and foreign consumption, when $v = 1$. It immediately follows that the own country fiscal spending multiplier must be greater than unity, since, in response to a rise in home government spending, home output must rise by more than $(1-c_y)\widehat{g}_t$.

Thus, we find that fiscal spending under a liquidity trap has a magnified multiplier, but this comes at the expense of a negative cross country multiplier. Intuitively, the impact of an increase in the spending gap in a liquidity trap is to raise expected inflation in the home country relative to the foreign country. Thus we can show that

$$E_t(\pi_{Ht+1} - \pi_{Ft+1})^{g}_{t(\text{ZLB})} = \frac{(1-c_y)(1-\mu)\mu\phi}{\Delta_4}(\widehat{g}_t - \widehat{g}^{*}_t).$$

Since the nominal interest rate is stuck at zero in both countries, then the real interest rate, in terms of domestic inflation, must *fall* in the home country relative to the foreign country. This requires an *anticipated* terms of trade appreciation, which must be facilitated by an immediate large terms of trade depreciation. The terms of trade depreciation increases demand for the home good, but reduces demand for the foreign good. Moreover, in this case, the latter effect offsets the impact of a rise in total aggregate consumption, so that foreign output must fall.

4.3 Open versus Closed Economy Fiscal Multipliers

There is an interesting contrast between the effects of fiscal policy shocks along the open-versus-closed economy dimension on the one hand, and the postive versus zero interest rate dimension on the other. To see this, begin first with the case where interest rates are positive and monetary policy follows a Taylor rule as before. Then the closed economy case is obtained from solving (29) and (30) under the restriction $v = 2$, (so that all households value only home domestic goods in consumption). In this case, the solution for (34) becomes

$$\widehat{n}^g_{t(\text{Taylor-closed})} = \frac{(1-c_y)((1-\beta\mu)(1-\mu)+\kappa(\gamma-\mu))\widehat{g}_t}{\Delta_1} \tag{39}$$

The difference between (34) and (39) is

$$\widehat{n}^g_{t(\text{Taylor})} - \widehat{n}^g_{t(\text{Taylor-closed})} =$$

$$-\frac{(1-c_y)k\phi c_y(\gamma-\mu)\,(\sigma-1)\,((1-\beta\mu)(1-\mu)+\kappa(\gamma-\rho))\widehat{g}_t}{\Delta_1\Delta_2} < 0$$

Which is always negative. That is, the open economy multiplier for government spending is lower than that of the closed economy, for a given monetary policy rule. This accords with textbook intuition about 'leakage' effects of fiscal policy shocks in the open economy. In this particular model, the reasoning can be tied to the behavior of the terms of trade. As we saw above, in an environment with monetary policy governed by a Taylor rule, a fiscal expansion causes a terms of trade appreciation. This dampens the demand effects of fiscal expansion on domestic GDP, and reduces the impact relative to that of a closed economy.

When interest rates are constrained by the zero lower bound, the distinction between the open and closed economy multipliers is very different. It is easy to see that in the closed economy at the zero lower bound the impact of fiscal policy is described as:

$$\widehat{n}^g_{t(\text{ZLB-closed})} = \frac{(1-c_y)((1-\beta\mu)(1-\mu)-\kappa\mu)\widehat{g}_t}{\Delta_3} \tag{40}$$

Taking this away from (38), we find that

$$\widehat{n}^g_{t(\text{ZLB})} - \widehat{n}^g_{t(\text{ZLB-closed})} =$$

$$\frac{(1-c_y)k\phi c_y\mu\,(\sigma-1)\,((1-\beta\mu)(1-\mu)+\kappa(\gamma-\rho))\widehat{g}_t}{\Delta_3\Delta_4} > 0$$

Thus, when both economies are in a liquidity trap, the open economy multiplier exceeds that of the closed economy. There is in effect 'reverse leakage'. As we have seen above, the impact of fiscal spending in the open economy under a liquidity trap is to cause a terms of trade deterioration. This drains demand away from foreign output and towards home output. The result is that foreign output falls, and home output rises by more than it would in a closed economy.

4.4 The general case with home bias in preferences

In the general case with home bias in preferences, the analysis of a liquidity trap becomes more complex. The central difference is that a shock to savings in one country may not push both countries into a liquidity trap at the same time. In particular, a home country preference shock which increases home savings will reduce the Wicksellian interest rate in the home country by more than that of the foreign country. Thus, the home country becomes constrained by the zero bound before the foreign country.

As a comparison, we first derive the output effects of government spending gap and preference shocks in the general case of home bias when both countries can freely follow a Taylor rule. This is simply a solution of the equations (29)-(32) to allow for the case $v > 1$. Define the variable $D = \sigma v(2-v) + (1-v)^2 > 1$. Then the analogous equations to (33) -(34) are[5]:

$$\widehat{n}^{r}_{t(\text{Taylor})} = \frac{(1-\mu)(1-\beta\mu)\phi c_y^2 \varepsilon_t}{(\sigma+\phi)\Delta_1} + \frac{D(1-\beta\mu)(1-\mu)c_y^2(v-1)\varepsilon_t}{2\Delta_1\Delta_5\sigma}$$

$$\widehat{n}^{*r}_{t(\text{Taylor})} = \frac{(1-\mu)(1-\beta\mu)\phi c_y^2 \varepsilon_t}{(\sigma+\phi)\Delta_1} - \frac{D(1-\beta\mu)(1-\mu)c_y^2(v-1)\varepsilon_t}{2\Delta_1\Delta_5\sigma}$$

where $\Delta_5 = (1-\beta\mu)(1-\mu) + k(1+\phi c_y \frac{DD}{\sigma})(\gamma-\mu) > 0$, and $\Theta = \frac{(\phi+\sigma)(1-v)}{v((\phi+\sigma+\phi c_y(2-v)(\sigma-1))} > 0$, where $\Theta = 1$, when $v = 1$.

Thus, a negative preference shock has a larger impact on home output relative to foreign output when there is home bias. In the extreme case $v = 2$, then effectively both economies are closed, and then since $D = 1$, and $\widehat{n}^*_t = 0$.

The impact of fiscal spending shocks in the general case, with active Taylor rules, may be described as (the effect on foreign output is symmetric):

$$\begin{aligned}\widehat{n}_t &= \frac{1}{2}(1-c_y)\frac{[2\Delta_1 + \kappa\phi c_y(D-1)(\gamma-\rho)]\left((1-\beta\mu)(1-\mu)+\kappa(\gamma-\rho)\right)\widehat{g}_t}{\Delta_1\Delta_5} \\ &\quad + \frac{1}{2}(1-c_y)\frac{[\kappa\phi c_y(D-1)(\gamma-\rho)]\left((1-\beta\mu)(1-\mu)+\kappa(\gamma-\rho)\right)\widehat{g}^*_t}{\Delta_1\Delta_5}\end{aligned}$$

Again, in the general case, fiscal spending shocks have positive impacts on home and foreign

[5]We write these equations in terms of the underlying demand shocks, ε_t, since in this case, the Wicksellian interest rate responses differ between the two countries.

output, so long as the Taylor rule applies.

When the shock to preferences is large enough that one or both economies are pushed into a liquidity trap, the analysis has to be redone. Since the case where both economies are in a liquidity trap is simply a routine extension of (38) to the environment with home bias, we concentrate on the asymmetric case, where the home economy suffers a savings shock that leads it into a liquidity trap, but the foreign economy still has a positive nominal interest rate.

Since the foreign economy can adjust its interest rate to offset the shock to $\widetilde{r}_t^*$, we have to make an assumption about the foreign monetary policy response. In the foreign monetary authority offsets $\widetilde{r}_t^*$, there is no direct macroeconomic shock to the foreign economy - the only way that the foreign economy is impacted is through movements in home output, and movements in the home fiscal spending gap. Generally, it is not obvious that it is an optimal policy for the foreign country to eliminate the interest rate gap in a global environment where the home country is constrained from doing so. In the next section, we will show that an optimal cooperative monetary and fiscal policy mix requires the foreign economy not to fully offset the shock to the Wicksellian interest rate. Here however, for the sake of clarity, we examine both cases - where the interest rate gap is closed by the foreign monetary authority, and where it is not.

The analytical expressions in the asymmetric outcome are relatively complex and give little intuition (see the Appendix for the exact expressions). Hence we illustrate the case of an asymmetric liquidity trap numerically. Tables 1 and 2 present the results for the government spending multipliers and the impact of savings shocks in the whole range of cases discussed so far.

We choose parameters as follows. Let $\beta = 0.99$, so that a period is a quarter in the standard calibration. The labour supply elasticity is set set at unity, so $\phi = 1$. We let $\kappa = 0.85$, so that $k = 0.027$, as in Rebelo et al. (2009). Let the share of government in output be 20 percent, so that $c_y = 0.8$. The coefficient on the Taylor rule is set at $\gamma = 1.5$. In addition, allowing for home bias in preferences, we let $v = 1.5$, so that the share of imports in steady state output is 20 percent[6]. Finally, we let the persistence of all shocks be set at 0.8 ($\mu = 0.8$). This implies that the liquidity trap is expected to last 5 quarters.

Before discussing the asymmetric case, we can briefly address the quantitative implications of the model in the absence of home bias. In the $v = 1$ case, where there is a symmetric liquidity trap in both countries, Table 1 shows that the impact of fiscal policy shocks on both the terms of trade and GDP are an order of magnitude higher when both

[6]This is higher than that for the US, but is a reasonable number for most OECD economies.

economies are stuck in a liquidity trap than under a Taylor rule. The Table illustrates the magnitude of government spending multipliers on GDP, as well as the impact of government spending shocks on the terms of trade, real home and foreign consumption, and the real interest rates in each country. When the Taylor rule applies, the government spending multiplier is approximately 0.8, while the cross country multiplier is 0.4. But when both economies are in a liquidity trap, the own multiplier is 4.5, while the cross country mutiplier is -2.8. The Table also shows that the impact of government spending shocks on the terms of trade in a liquidity trap is an order of magnitude greater, *and with the reverse sign*, than under the Taylor rule. The key intuition behind these results is contained in the bottom row of Table 1 - a government spending expansion under a liquidity trap leads to a fall in real interest rates in each country, compared to a (small) increase in real interest rates under a Taylor rule.

A similar set of conclusions applies to the impact of savings shocks. Under a symmetric liquidity trap, the impact of savings shocks on output in both countries is much larger than under a Taylor rule, and this can be ascribed to the perverse effects of the shock on real interest rates.

Returning now to the baseline calibration, with $v = 1.5$, we can observe the impacts of a savings shock as well as the government spending multipliers when either one or both countries is constrained by the zero lower bound. Column 3 of Table 2 describes the situation where both economies are in a liquidity trap. The comparison between the third columns of Table 2 and Table 1 thus illustrate the influence of home bias on the the response to shocks under a symmetric liquidity trap. The greater is home bias, the more each country resembles a closed economy. As we have seen analytically above, this leads to a smaller own-country government spending multiplier in a liquidity trap, and also reduces the absolute size of the negative cross country spending multiplier. The response of the terms of trade is analogous. With greater home bias, the response of the terms of trade to government spending shocks is reduced.

While openness tends to exacerbate the impacts of government spending shocks under a liquidity trap, the opposite reasoning applies to the response to savings shocks. A comparison of columns 3 of Table 1 and 2 show that openness tends to cushion the output response to shocks to preferences. In response to an exogenous increase in home country savings, home output falls almost twice as much in the economy with home bias than in the case with symmetric preferences. In addition, home bias in preferences actually *reverses* the impact of preferences shocks on the foreign economy. As home bias magnifies the response of the home economy to a saving shock, it leads to a much larger movement in relative prices. A negative savings shock which causes a large reduction in home output leads to a big terms of trade appreciation, and from Table 2 we see that this actually increases foreign output. Thus, in the realistic case of home bias in preferences, we find that a liquidity trap tends to generate asymmetric spillovers of savings shocks as well as government spending shocks.

Finally, we can analyze the impact of an asymmetric liquidity trap outcome. Columns 4 and 5 illustrate the behaviour of the home and foreign economies when the home economy is constrained by the zero bound, while the foreign economy operates freely under a Taylor rule. Column 4 is based on the assumption that the foreign monetary authority does not offset the direct effect of the preference shock on the foreign Wicksellian interest rate, while column 5 assumes that it does offset the shock. We see that the response to the preference shock is almost the same in each case. The central comparison is between column 3 and column 4, which illustrate the difference between the response of variables in the global liquidity trap to the under a liquidity trap in the home country only. The key difference is the asymmetry in international spillovers of fiscal policy shocks. Government spending shocks in the foreign economy have a positive multiplier effect (but very small) on the home economy, while home country government spending shocks have a substantial negative spilllover on the foreign economy. A similar asymmetry applies to the response of the terms of trade. The terms of trade are significantly more sensitive to government spending shocks in the home country, i.e. the country in which there is no endogenous interest rate response. An interesting feature about this case (the asymmetric liquidity trap case) is that government spending shocks in both economies tend to generate terms of trade depreciation - i.e. the effects in both the home and foreign economies are of the same sign.

5 Optimal Monetary and Fisal Policy

In this section, we return to the questions raised in the introduction. What is the optimal coordinated response to a global liquidity trap, and what role does fiscal policy have in complementing the response of monetary policy? We focus on the optimal cooperative outcome for two reasons. First, it sets out a benchmark for choosing a policy response to as to maximize world welfare in response to a negative demand shock which undermines the normal mechanisms of monetary policy. Secondly, the alternative scenario, where governments and monetary authorities act independently so as to maximize national welfare, raises thorny issues of dynamic strategic interaction which cannot easily be handled within the current framework.

Hence we proceed as follows. Assume that at date t there is a negative demand shock to home country preferences, as before. Again, assume that this shock remains in effect with probability μ in each period, and disappears with probability $1-\mu$. Assume that the shock is large enough that at least the home country would be required to reduce nominal interest rates below zero so as to eliminate the effect of the shock on inflation and the output gap. As before, in the case $v=1$, this must be also true of the foreign country. But in general, the foreign country may or may not be pushed down to the zero lower bound.

5.1 Optimal Monetary and Fiscal Policy in the case $v = 1$

We model the optimal monetary and fiscal policy response as follows. In the appendix, it is shown that the global equally weighted loss function for the authorities, up to a second order approximation, may be written as a quadratic function of output gaps, government spending gaps, inflation rates, and cross products. In the symmetric case, without home bias, the one-period welfare function is expressed as:

$$\begin{aligned} V_{(v=1),t} &= -(\widehat{n}_t - \widehat{n}_t^*)^2 \frac{(1+\phi c_y)}{2c_y^2} - (\widehat{n}_t + \widehat{n}_t^*)^2 \frac{(\sigma + \phi c_y)}{2c_y^2} \\ &\quad -(\widehat{g}_t - \widehat{g}_t^*)^2 \frac{(1-c_y)((1-c_y)+c_y\sigma)}{2c_y^2} - \frac{(1-c_y)}{2c_y^2}(\sigma(\widehat{g}_t + \widehat{g}_t^*)^2 \\ &\quad -2((\widehat{n}_t - \widehat{n}_t^*)(\widehat{g}_t - \widehat{g}_t^*) + \sigma(\widehat{n}_t + \widehat{n}_t^*)(\widehat{g}_t + \widehat{g}_t^*)) - \frac{\theta}{k}\pi_{Ht}^2 - \frac{\theta}{k}\pi_{Ft}^2 \end{aligned} \tag{41}$$

In this evaluation, we have assumed that the interemporal elasticity of substitution in government consumption goods is equal to that in private consumption goods. This imposes a welfare cost of opening a gap between the actual and efficient flexible price optimal level of public spending.

We first examine the optimal combination of monetary and fiscal policy in when both countries are in a liquidity trap simultaneously. We focus (for now) on purely discretionary policy. That is, the cooperative policy chooses inflation rates, the output gap and government spending taking future values of these variables as given, subject to the fact that both economies are simultaneously in a liquidity trap, but with probability $1-\mu$, both will exit the liquidity trap in the next period.

The policy problem under discretion can be described as one of maximizing (41) subject to the two inflation equations (29) and (31), and the two conditions $r_t = 0$, and $r_t^* = 0$ from (30) and (32). We may write the Lagrangean as

$$\text{Max}_{\left[n_t, n_t^*, \pi_{Ht}, \pi_{Ft}^*, g_t, g_t^*\right]} \quad V_{v=1,t}$$

$$+\lambda_1\left(\pi_{Ht}-k(\phi\widehat{n}_t+\tfrac{\sigma}{2c_yD}\left[\widehat{ng}_t(1+\sigma)+\widehat{ng}_t^*(\sigma-1)\right]-\beta E_t\pi_{Ht+1}\right)$$
$$+\lambda_2\left(\pi_{Ft}-k(\phi\widehat{n}_t^*+\tfrac{\sigma}{2c_yD}\left[\widehat{ng}_t^*(1+\sigma)+\widehat{ng}_t(\sigma-1)\right]-\beta E_t\pi_{Ft+1}\right)$$
$$\psi_1E_t\left(\tfrac{\sigma}{c_y}\left((\widehat{ng}_{t+1}-\widehat{ng}_t)+E_t(\widehat{ng}_{t+1}^*-\widehat{ng}_t^*)\right)+\pi_{Ht+1}+\widetilde{r}_t+\tfrac{\left((\widehat{ng}_{t+1}-\widehat{ng}_{t+1}^*)-(\widehat{ng}_t-\widehat{ng}_t^*)\right)}{2}\right)$$
$$\psi_2E_t\left(\tfrac{\sigma}{c_y}\left((\widehat{ng}_{t+1}-\widehat{ng}_t)+E_t(\widehat{ng}_{t+1}^*-\widehat{ng}_t^*)\right)+\pi_{Ft+1}+\widetilde{r}_t-\tfrac{\left((\widehat{ng}_{t+1}-\widehat{ng}_{t+1}^*)-(\widehat{ng}_t-\widehat{ng}_t^*)\right)}{2}\right).$$

Here, λ_1 and λ_2 are the Lagrange multipliers on the inflation equations and ψ_1 and ψ_2 are the multipliers on the nominal interest rate constraints. Notice that, in line with the previous section, in the symmetric case with complete markets, both countries enter the liquidity trap simultaneously.

Choosing the combination of inflation rates, output gaps, and government spending gaps to solve this problem under discretion leads to the following optimal policy rule for the government spending gap:

$$\widehat{g}_t=-\widetilde{r}_t\frac{\phi c_y((1-\beta\mu)+k\theta(\sigma+\phi c_y))}{\Delta_6} \tag{42}$$

where the denominator is defined as:

$$\begin{aligned}\Delta_6 &= \sigma((1-\beta\mu)(1-\mu)-k\mu)+\\ &\quad \phi((1-\beta\mu)(1-\mu)(1-c_y)-\kappa\mu)+c_y\phi k\theta\,(1-\mu)\,(1-c_y))\end{aligned}$$

Under the stability conditions, this denominator is positive. Hence government spending is negatively related to the movement in the Wicksellian interest rate in an optimal cooperative fiscal policy plan. Because countries are identical and symmetric, foreign government spending obeys the same condition.

Under the optimal fiscal plan, the output gap is identical in each country, and satisfies the condition:

$$\widetilde{n}_t=-\frac{\sigma(1-\beta\mu)-\theta k\phi(1-c_y)\widetilde{g}_t}{((1-\beta\mu)+k\theta(\sigma+\phi c_y))}$$

Hence, and optimal fiscal policy plan in a liquidity trap is countercyclical. When the economy is pushed into a liquidity trap through a fall in the Wicksellian interest rate, the output gap falls, but government spending responds positively - government spending increases above the optimal, flexible price efficient level of government spending, so as to stabilize the economy. Moreover, this is a coordinated (identical) pattern of government spending in each country.

How much does fiscal policy contribute to extracting the economy from a liquidity trap-induced recession? If the fiscal spending gaps were constrained to be zero, then in the case of monetary policy discretion, the output gaps are determined directly by the combination of the forward looking inflation equations and the zero bound conditions on nominal interest rates. The output gaps are then:

$$\widehat{n}_t = -\widetilde{r}_t \frac{c_y(1-\beta\mu)}{\Delta_4}$$

This is always greater in absolute terms that when fiscal policy is employed. An interesting additional feature of (42) is that active fiscal policy is employed in response to demand shocks, even as the policy weight on inflation rises significantly. This is because it is possible to pursue active fiscal policy without inflationary consequences, so long as the expansion does not impact on marginal cost.

It is easy to show that this optimal government spending rule is identical to that which would apply in a closed economy. Although the individual government spending multipliers are quite different than those in a closed economy, as shown in the previous section, the added impact on 'own spending' and the negative effect of 'others spending' cancel out in the optimal policy decision, so that the optimal spending response in the case of pure symmetry is the same as in a closed economy. This is reminiscent of Clarida et al's (2002) result that in an open economy with a substiution elasticity of unity across home and foreign goods and no home bias, the monetary policy problem is parallel to that of a closed economy.

5.2 The General Case

Now we extend the model to accomodate home bias in preferences, and examine the optimal policy problem in this setting. When $v > 1$, the welfare function has to be further expanded from (41). Again, the appendix shows that the welfare function for the cooperative policy problem is given by:

$$V_t = V_{(v=1),t} - \frac{1}{2}(\widehat{n}_t - \widehat{n}_t^*)^2\Gamma_1 - \frac{1}{2}(\widehat{g}_t - \widehat{g}_t^*)^2\Gamma_2 + ((\widehat{n}_t - \widehat{n}_t^*)(\widehat{g}_t - \widehat{g}_t^*)\Gamma_3 \tag{43}$$

where $\Gamma_1 = \frac{(v-1)^2(\sigma-1)}{D}(1 + \frac{(1-c_y^2)}{c_y^2 D})$, $\Gamma_2 = \frac{(1-c_y)^2(v-1)^2(\sigma-1)}{c_y^2 D^2}(1 + v(v-1)(\sigma-1)(2-v)c_y^2)$, and $\Gamma_3 = \frac{(1-c_y)(v-1)^2(\sigma-1)}{c_y^2 D^2}(1 + v(v-1)(2-v)(\sigma-1)c_y^2)$.

There are two main cases to consider in this extended environment. In one case, the demand shocks are small enough that only the home economy is pushed into a liquidity trap. In the second case, the shocks are sufficiently large that both economies are pushed down to the zero lower bound. We treat each case in turn.

The analytical description of the optimal policy problem in the asymmetric case is rather complex. Hence, we describe the optimal policy outcome numerically. Table 3 lists the results of the optimal policy rules under discretion, first in the symmetric case just discussed, and then in the case of home bias, both when only the home economy is constrained by the zero bound, and when both economies are constrained together. We use the same numerical calibration as described in the previous section. In addition, we set the elasticity θ to 5, a standard parameter value in the literature. In the calibration described above, the quarterly nominal interest rate in a steady state is $1/\beta - 1 = .01$. In the symmetric case, we assume that the preference shock in the home country is such that the quarterly value for the Wicksellian real interest rate is pushed down to -0.01, and as stated previously, this shock persists with probability μ in each successive period. In this case, both countries are in a liquidity trap. In the case $v = 1.5$, we take make two alternative assumptions. We first assume that the preference shock is large enough that both economies are pushed into a liquidity trap. Specifically, we assume that the Wicksellian interest rate in the home economy is pushed down to -0.035 per quarter, which given the calibration implies the foreign economies real interest rate would be -0.003. Hence, the home economy is deep in a liquidity trap, while the foreign economy is just barely in a trap. We then look at a smaller shock which only puts the home economy into a liquidity trap, requiring a Wicksellian interest rate of -0.027 per quarter, with the foreign economy desired real interest rate just above zero.

Table 3 illustrates the characteristics of optimal policy when fiscal policy is inoperative, and when the optimal coordinated fiscal policy is used. In the case $v = 1$, (described analytically above), without fiscal policy there is no active policy at all, since neither country can use monetary policy for the duration of the liquidity trap. In this case, without the fiscal spending response, output in both countries falls by 2.8 percent, and there is a deflation rate of -.4 percent. The optimal active fiscal policy is to raise the spending gap by 2 percent, which is equivalent to a rise in government spending of about 0.4 percent of GDP, and this limits the fall in output to 2.4 percent, and limits the rate of deflation to .37 percent. In this symmetric case, there are no consequences for the terms of trade, since in response to the home country preference shock, with complete markets, home and foreign output move in tandem, whether fiscal policy is set optimally or not.

In the case with home bias, the second column of Table 3 shows the case in which both countries are in a liquidity trap. But because we are examining an initial savings shock that originated in the home country, the consequences for home GDP are far greater. In the absence of fiscal policy, output falls by 7.3 percent in the home country, but only by just over 1 percent in the foreign country. In addition, the rate of deflation is 1 percent in the home country but only .3 percent in the foreign country. that the two countries behave very differently. The fiscal policy response likewise calls for a substantially higher increase in the home spending gap - by 4.8 percent, relative to an increase of only 1 percent for the foreign country. This reduces the output fall by about 1 percent in the home country, and by about 0.2 percent in the foreign country.

Finally, column 3 of Table 3 describes the asymmetric case, where the home economy is in a liquidity trap, but the foreign economy is not. Again, without the use of fiscal policy, the home economy has a much greater fall in GDP. In this case, an active fiscal policy involves both countries increase their government spending gaps - but the desired increase is much greater for the home economy. Note that this is a case where the foreign country has an unconstrained monetary policy, so it is free to fully offset the shock to the Wicksellian interest rate. Nevertheless, it still participates in a fiscal expansion as part of the optimal coordinated policy.

Because the foreign economy has a free monetary policy in this case, it can adjust interest rates in response to the shock to the desired real interest rate. The lower panel of Table 3 describes the response of the foreign nominal interest rate. The foreign country does reduce interest rates, but only by about a third of the actual fall in the foreign Wicksellian interest rate.

How much of an overall contribution does fiscal policy make in responding to the negative savings shock? Table 3 indicates that expansionary fiscal policy is used, but it is not nearly large enough to eliminate the output gaps generated by the original savings shock in any of the cases. Since generating fiscal spending gaps has a utility cost in itself, it is never optimal to close all gaps with fiscal expansion.

How much of a contribution does foreign fiscal policy make in the optimal coordinated response? We also extended to the asymmetric liquidity trap case in Table 3 to the case where the foreign country was restricted to maintain its fiscal spending gap equal to zero. We found that the equilibrium home country fiscal policy response was now higher, but only by a miniscule amount, and the equilibrium output gap is also very slightly higher. But essentially, the response of output and inflation in both economies is extremely close to the case where foreign fiscal policy is unconstrained. Hence, an optimal fiscal policy response to a domestic liquidity trap is almost entirely taken up by adjustment in domestic policy, and elicits essentially no adjustment in foreign policy.

5.3 Conclusions

To be written..

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	Table 1 Case v=1					
	Flexible Prices			Taylor Rule		
Variable	g	g*	ε	g	g*	ε
Terms of Trade	-0.1	0.1	0	-0.05	0.05	0
Home GDP	0.6	0.08	0	0.84	0.04	0.033
Foreign GDP	0.08	0.6	0	0.04	0.84	0.033
Home Consumption	-0.036	-0.036	0.25	-0.14	-0.14	0.29
Foreign Consumption	-0.036	-0.036	-0.25	-0.14	-0.14	-0.21
Home Real IR	0.014	0.014	0.03	0.01	0.01	0.01
Foreign Real IR	0.014	0.014	0.03	0.01	0.01	0.01
	Global Liquidity Trap					
Variable	g	g*	ε			
Terms of Trade	1.6	-1.6	0			
Home GDP	4.5	-2.8	0.2			
Foreign GDP	-2.8	4.5	0.2			
Home Consumption	0.1	0.1	0.49			
Foreign Consumption	0.1	0.1	0.49			
Home Real IR	-0.04	-0.04	-0.07			
Foreign Real IR	-0.04	-0.04	-0.07			

Notes: This table derives the impacts of Home government spending, foreign government spending, and a home country demand shock on the various variables, in the case where preferences are identical across countries (so the liquidity trap must be simultaneous). The entries denoted Home GDP and Foreign GDP define the multiplier effects of government spending, but the percentage response to demand shocks. In a Taylor rule, both economies adjust interest rates to inflation as described in the text. In a global liquidity trap, neither country adjusts interest rates for the duration of the liquidity trap.

	Table 2 Case v=1.5					
	Flexible Prices			Taylor Rule		
Variable	g	g*	ε	g	g*	ε
Terms of Trade	-0.12	0.12	0	-0.05	0.05	.22
Home GDP	0.65	0.06	0	0.85	0.03	0.05
Foreign GDP	0.06	0.65	0	0.03	0.85	0.001
Home Consumption	-0.05	-0.05	0.25	-0.02	-0.01	0.26
Foreign Consumption	-0.05	-0.05	-0.25	-0.01	-0.02	-0.18
Home Real IR	0.02	0.01	0.02	0.01	0.001	0.004
Foreign Real IR	0.01	0.02	0.03	0.001	0.01	0.017
	Global Liquidity Trap			Home Liquidity Trap		
Variable	g	g*	ε	g	g*	ε
Terms of Trade	0.89	-0.89	0.66	0.5	0.07	0.57
Home GDP	2.19	-1.17	0.53	2.42	0.09	.5
Foreign GDP	-1.17	2.9	-.14	-.34	0.84	-.1
Home Consumption	0.21	-0.02	0.58	0.2	-.001	0.58
Foreign Consumption	-0.02	0.21	-0.09	0.07	-0.02	0.07
Home Real IR	-0.08	-0.001	-0.12	-0.08	-0.001	-0.12
Foreign Real IR	-0.001	-0.08	-0.02	-0.03	-0.08	-0.03
	Home Liquidity Trap (foreign offset)					
Variable	g	g*	ε			
Terms of Trade	0.5	0.07	0.63			
Home GDP	2.42	0.09	.52			
Foreign GDP	-.34	0.84	-.12			
Home Consumption	0.2	-.001	0.58			
Foreign Consumption	0.07	-0.02	0.08			
Home Real IR	-0.08	-0.001	-0.12			
Foreign Real IR	-0.03	-0.08	-0.04			

Notes: This table derives the impacts of Home government spending, foreign government spending, and a home country demand shock on the various variables, in the case with home bias in preferences. The entries denoted Home GDP and Foreign GDP define the multiplier effects of government spending, but the percentage response to demand shocks. In a Taylor rule, both economies adjust interest rates to inflation as described in the text. In a global liquidity trap, neither country adjusts interest rates for the duration of the liquidity trap. In the home liquidity trap, the foreign economy adjusts interest rates as in a Taylor rule. In the last case, we also let the foreign economy offset the shock to the flexible price real interest rate.

Table 3 Case v=1.5			
No Fiscal policy	V=1	V=1.5 (Global Liquidity Trap)	V=1.5 (Home Liquidity Trap)
Home GDP	-0.028	-0.073	-0.046
Foreign GDP	-0.028	-0.011	.0.002
Home Consumption	0	0	0
Foreign Consumption	0	-0	0
Home Inflation	-0.004	-0.0089	-0.0053
Foreign Inflation	-0.004	-0.003	-0.001
Active Fiscal policy	V=1	V=1.5 (Global Liquidity Trap)	V=1.5 (Home Liquidity Trap)
Home GDP	-0.024	-0.063	-0.04
Foreign GDP	-0.024	-0.009	.0.001
Home Consumption	0.02	0.048	0.03
Foreign Consumption	0.02	-0.01	0.003
Home Real IR	-0.0037	-0.0084	-0.0051
Foreign Real IR	-0.0037	-0.0028	-0.001

Notes: This table derives the results of the optimal coordinated package of monetary and fiscal policies when both countries maximize the world social welfare index given in the text, and there is home bias in preferences. In the top panel, fiscal policy in both economies is fixed to eliminate the fiscal gaps only. In the bottom panel, an optimal active fiscal policy is used. In the second column, both countries are in a liquidity trap, while in the third column, only the home economy is in a liquidity trap. The magnitude of the shocks which precipitate the liquidity trap is stated in the text.