The Empirical Study on Long-Memory Properties of Korean Won/Dollar Exchange Rate and Volatility

Ha-Hyun Jo* and Seungkook Lee**

This paper examines the existence of long-memory properties in Won/Dollar exchange rate and its volatility. Understanding DGP (Data Generating Process) of time series helps to specify a prediction model correctly and, further, it is an important bridge to constructing useful trading strategies.

Unlike conventional unit root tests that differentiate between I(0) and I(1) process, recent econometric work has been focusing on figuring out long-memory properties based on fractional differencing methods initiated by Granger et al.(1980).

Therefore, this study reviews several detection methods for long memory structure (or long-range dependence) in time series, and tests the time series properties of won/dollar exchange rates by using those methods. Furthermore, in order to identify a long-memory property in the volatility of exchange rates (exrates hereinafter), we used three volatility models, namely, Historical Volatility, EWMA, and GARCH model.

According to our empirical test, the won/dollar exrates and volatility both have long memory properties, and the properties appear more clearly in the latter. These results indicate that in constructing the prediction model for exrate volatility, FIGARCH model can be preferred to GARCH model because FIGARCH provides a more efficient way to represent the long memory properties.

* Dept. of Economics, Yonsei University (E-mail: hahyunjo@dreamwiz.com)
** Dept. of Economics, Yonsei University (E-mail: sklee89@dreamwiz.com)

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Ⅰ. Introduction

Traditional financial theories assume that asset returns follow a random walk process, thus normally distributed. Under such assumption, many linear stochastic models had been developed to explain asset prices movements. However, recent empirical studies illustrate that representative financial variables follow a Biased Random Walk (BRW) indicating a persistent trend, not a random walk. Accordingly, much interest is being focused on non-normal distribution, in particular. leptokurtic distribution with fat-tails in describing several important characteristics of asset prices.

The ReScaled analysis (R/S) pioneered by Hurst(1951), a hydrologist, is one of the methods to detect long memory (LM) properties in time series. The Hurst exponent derived from R/S analysis is related to a Fractional Brownian Motion (FBM), provided by Mandelbrot(1969) and Peters(1991, 1994).[1)]

In Hong(1998)'s empirical work on the long memory (LM) properties of Korean stock prices and interest rates, he rejected the hypothesis of the existence of LM properties by using R/S analysis. On the other hand, Kim(1992), Kim(1995), and Kim-Kang(1991), using a spectrum analysis and Cochrane's variance ratio test, showed that the effect of shocks on exchange rates disappeared very slowly.

ARIMA(p,d,q) is a standard model for specifying a short-range dependence, where d must be an integer. As a more generalized model of ARIMA, ARFIMA(p,d,q) has the ability to capture a long-range dependence (LRD) properties of exogenous shocks efficiently, as well as short-range dependence. The d in ARFIMA model can be a noninteger, thus it is called a fractional difference parameter. The ARFIMA can represent a very slowly decaying Autocorrelation Function (ACF) that is often observed in financial time series.

ARFIMA approach recognized as an alternative method to the R/S analysis in analyzing long memory properties, and the works on similarities and differences

Notes : 1) A FBM is a continuous version of the Biased Random Walk (BRW).

between these two are still on-going.

The objectives of this paper are to identify the existence of long memory (LM) or long-range dependence (LRD) properties in the Korean won/dollar exrates and its volatility data. If the existence of the properties could be verified empirically, it implies that the canonical paradigm based on normality/random walk hypothesis had better be replaced. Instead, the situation calls for an analytical framework based on leptokurtosis/BRM.

That is, previous financial theories under normality assumption, such as $T^{1/2}$-rule, need to be re-examined, if financial variables do not follow a normal distribution. Moreover, prediction models for financial variables must be specified on the basis of long memory properties.

This paper is organized as follows.

Section Ⅱ reviews some concepts of long memory properties. Then it compares an LM called the Hurst phenomenon with the LRD as represented in slowly decaying autocorrelations. Though much of the earlier literature had not distinguished these two properties, our study do consider the LM as a intrinsic feature of time series, and the LRD as an exogenous shock's long-run persistency.

Section Ⅲ describes the empirical test methodologies for detecting long memory properties. Next, section Ⅳ provides the test results of the existence of LM/LRD in the Korean won/dollar exrates, and its' volatility series measured by three popular models (Historical Volatility, EWMA, GARCH). Finally, section Ⅴ summarizes the test results, and then presents the practical implications with regard to financial market analysis.

Ⅱ. Understanding LM/LRD

1. Hurst phenomenon and Long Memory (LM)

The study of long memory properties was pioneered by Hurst(1951)'s works on the Nile dam's reservoir. In order to reflect the seasonality of river flow, the storage capability of the dam should cover the variability of the influx from the river. In particular, regular outlays of riverflow were required to prevent a drought or flood. And the estimation of a long-run reservoir of a dam is very important work because the construction cost of a dam is very high.

If the riverflow series would display a high persistence, the required reservoir measured by range should increase accordingly.

In order to measure the degree of persistence, Hurst(1951) developed the R/S statistic, a nonparametric estimation technique. He then applied it to a number of natural phenomena such as rainfall, sunspots and He found that many natural time series had long memory structures as an intrinsic feature.

The Hurst exponent (H) derived from R/S analysis helped to figure out the degree of correlation among observations. Specifically, a correlation measure (C) can be calculated as $2^{(2H-1)}-1$ from the Hurst exponent.[2)]

As shown above, if , $H = 0.5$, i.e., the correlation measure would be 0, which means the series follows a random process. When $0 < H < 0.5$, i.e., it has a negative correlation measure, then the series is an anti-persistent and mean-reverting process. When $0.5 < H \leq 1$, the process is a persistent process, i.e., long memory. The degree of the persistence or long memory will increase as the Hurst exponent approaches 1, with the correlation measure presenting high positive values. Such time series are said to follow a Biased Random Walk (BRW) or Fractional Brownian Motion (FBM).[3)]

These Hurst phenomena related to fractal structures which was documented by Mandelbrot(1972), are applied to financial market analysis by Peters(1991). As mentioned above, although certain time series have the same means and variances, they can be classified into distinct processes according to the degree of persistence. The traditional financial literature, however, focused only on mean and variance (up to the second moment), not on the long memory property. Therefore, if financial variables have long memory structures, it implies we should not ignore other statistical features of financial time series as well as means and variances.

2. Long Range Dependence (LRD)

The Hurst exponent was developed for detecting long memory structures as an intrinsic feature, while most recent econometric work has emphasized the persistence of exogenous shocks. The previous studies on persistence were mainly implemented using unit root tests, and developed into a concept of cointegrated relations. The criticism on such a razor edge distinction between I(0) and I(1) process has been raised because the stream of research can provide

2) Here, a correlation measure(C) does not mean a simple autocorrelation function, it just means the degree of dependence generally.

3) Lo(1991) noted that the anti-persistent process ($H < 0.5$) has also long memory property. But, our empirical study, though not reported, supported Lo's view as several chaotic functions show anti-persistence. However, our main interests lie in persistent processes, so we will regard the long memory property as $H > 0.5$.

limited results.

On the light of this critique, Granger-Joyeux(1980), and Hosking(1981) proposed fractionally integrated ARMA model (ARFIMA) having the features of a slowly-decaying autocorrelation function (ACF) and the long-run persistence of shocks.

In ARFIMA model, the difference parameter (d) need not be an integer. This model is considered as a more general form than ARIMA, which can capture a hyperbolically decaying ACF including exponential decaying ACF in I(0), and infinite persistence in I(1) process.

To understand the ARFIMA model, firstly, consider a typical ARIMA(p,d,q) model.

$$\Phi(L)(1-L)^d Y_t = \Theta(L)\varepsilon_t,\ \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \tag{1}$$

where, $\Phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p$, $\Theta(L) = 1 - \theta_1 L - \ldots - \theta_q L^q$, the roots of $\Phi(L)$ and $\Theta(L)$ are assumed to exist outside of the unit circle.

Under a stationary ARIMA(p,d,q), d would be 0, while it would be 1 if there exists a unit root. Although ARFIMA(p,d,q) model is similar to ARIMA model, a difference parameter (d) is no longer restricted to integer.

The stationarity and invertibility condition in an ARFIMA model is $|d| < 0.5$. In the region of $|d| < 0.5$, the ACF can be obtained by k^{2d-1} (k : lag), which displays a hyperbolic decay as lag increases.

To test an LRD property in ARFIMA, we only need to estimate difference parameter (d). If the estimated d is above 0, the process has LRD. If $d < 0$, the process possesses a mean-reverting property. Suppose the estimated d is larger than 0.5, then the process is considered as nonstationary having an infinite variance. The usefulness of the fractional differencing technique is that we can figure out the degree of persistence concretely according to the estimated magnitude of d.

As indicated in [Table 1], the 1st ACF of ARFIMA ($d = 1/3$) and AR(1) is same as 0.5, but in lag 25, the ACF of AR(1) decays sharply to 2.98×10^{-8}. In contrast, the ACF of ARFIMA remains 0.173, and the value of 0.109 when the lag is 100. Thus, the model can display very slowly decaying autocorrelations.

3. Long Memory vs Long Range Dependence

The long memory (LM) property like the Hurst phenomenon regards irregular

Table 1 **Comparison of ACF**

lag (k)	ACF (d = 1)	ACF (d = 1/3)	ACF (AR(1))*
1	0.995	0.500	0.500
2	0.991	0.400	0.250
3	0.987	0.350	0.125
4	0.982	0.318	0.063
5	0.978	0.295	0.031
10	0.955	0.235	0.001
25	0.879	0.173	2.98×10^{-8}
50	0.785	0.137	8.88×10^{-16}
100	0.628	0.109	7.89×10^{-31}

* AR(1) : $x_{t+1} = 0.5x_t + \varepsilon_t$
* source : Lo(1991), [Table 1]

behaviors of time series as having the intrinsic nature of nonlinear dynamics, rather than exogenous shocks. In particular, time-varying volatility, a key feature of financial time series, is considered as a byproduct of the LM structure. Hence, in order to construct useful trading strategies, financial market participants should identify factors affecting LM structures.

For example, traders with long investment horizons wait until a clear price trend is revealed for decision-making. If the proportion of long-run traders within a market increases, asset prices would show a more persistent trend.

In modeling LM structures, however, attention should be paid to clarifying LM differently from LRD. An LM structure could be made from endogenous interaction among several factors within a financial market system. On the other hand, the LRD structure is generated from the long-run effect of exogenous shocks. Unfortunately, it is very difficult to divide the properties in an actual case.

In case of LRD, it is possible to directly model it as ARFIMA(p,d,q) while it is not a simple task to model LM features since its functional forms are unknown. Typically, despite chaotic functions having these structures, it is also possible for other functional forms to possess LM structure. Further, the relationship between LM and chaos is uncertain.[4)]

Possible links between LM and LRD could be described as follows.

A process with $H = 0.5$ is I(0), meaning its ACF decays exponentially. In such

4) The methods - difference parameter estimation, R/S analysis - for detecting long memory structures in time series had several problems. Notably, when the process has a nonlinear structure, the estimation technique of d in ARFIMA(p,d,q) has a critical drawback because this approach assumes implicitly that the DGP of time series is in a linear form. Also, Modified R/S analysis is known to be unable to identify some LM processes from deterministic chaos.

Table 2 LM vs LRD

	LM	LRD
Testing method	- R/S analysis - Modified R/S analysis	Estimating difference parameter (d) - Geweke, Porter, Hudak method (GPH) - Fox-Taqqu method - Sowell method - Wavelet method
Features	Intrinsic & nonlinear dynamics	exogenous shock
Modeling	- Chaos - Non-chaotic models	- ARFIMA (mean) - FIGARCH (variance)

sense, it could be referred to as a random process with only short memory. A process with $0.5 < H < 1$ indicates that it has LM property with a persistent trend, which corresponds to $0 < d < 0.5$. An anti-persistent process with $0 < H < 0.5$ results in $-0.5 < d < 0$.

Therefore, the theoretical relation between H and d is as follows.

$$H = d + \frac{1}{2} \tag{2}$$

where, H : Hurst exponent, d : the difference parameter.

Ⅲ. Research Methodology

1. Classical R/S

In order to resolve the problem of control of the Nile dam's reservoir, Hurst(1951) measured how the reservoir level fluctuated around its average level over time. The range of this fluctuation would change depending on the length of the time period used for measurement. If the process was random, the range would increase with the square root of time, i.e., the so-called $\sqrt{T}$-rule.[5)]

Hurst developed ReScaled range analysis (R/S analysis), a dimensionless ratio by dividing the range by the standard deviation of the observations. Through empirical work, he found that many time series from natural phenomena

5) Originally, Einstein documented this rule in measuring the distances among particles following Brownian motion. In finance, this rule is employed to expand a one-period volatility to a multi-period volatility. i.e., to get a return's standard deviation for one year, multiply a standard deviation for a month by $\sqrt{12}$.

followed a BRW process, not a random walk process.

To describe R/S analysis, we begin with cumulative deviations from a sample mean over N period, where N means the subsample size.

$$X_N = \sum_{i=1}^{N}(e_i - M_N) \tag{3}$$

where, X_N : cumulative deviation over N,
e_i : individual realizations, $i=1, ..., N$.
M_N : sample mean over N.

The number of subsamples with full sample size T is k $(=T/N)$, and the variation range can be calculated as follows :

$$R \equiv Max(X_N) - Min(X_N) \tag{4}$$

where, R : range of X,
$Max(X_N)$: maximum value of X,
$Min(X_N)$: minimum value of X.

The range measured by R may vary according to the time length (i.e., the size of N) ; hence calculated R needs to be divided by the standard deviation (S) for standardization.

The Rescaled range can be formulated by Hurst's empirical rule as follows :

$$R/S = (ak)^H \tag{5}$$

where, R/S : Rescaled range,
k : number of subsamples $(\frac{T}{N})$,
a : constant,
H : Hurst exponent.

The Hurst exponent (H) can be estimated by equation (5), which indicates the degree of shock persistence.

$$\log(Q_k) = constant + H\log(k) \tag{6}$$

where, Q_k : Rescaled range (R/S).

For eq.(6), we can get the estimate of Hurst exponent (H) by the OLS method.

As discussed earlier, H would equal 0.5 if the series is a random process. With $H>0.5$, the process has a long memory structure.

Though the classical R/S analysis has an advantage of being free of critical assumptions on the series distribution, it may be biased if the process has a short-range dependence. It also has the defect that it does not have a formal test statistics.

2. Modified R/S

As Lo(1991) pointed out, if a time series have a short-range dependence, the Hurst exponent could be upward biased. With a time series having a short-range dependence, judging the existence of LM by estimated H may lead to wrong inferences.

Notably, the Korean currency market is more likely to have short-range dependence due to several institutional restrictions or the central bank's interventions. Thus the classical R/S analysis for testing LM properties might generate a biased estimator.

Considering this problem, Lo(1991) presented the so-called "Modified R/S test statistics", which can test the long memory properties after eliminating the upward bias that the short-range dependence might cause.[6)]

He provided a distribution table for testing whether an analyzed series has LM or not, even after adjusting for a short-range dependence. In case of the non-existence a short-range dependence, V, the limiting distribution of $Q/\sqrt{T}$, is as follows.

$$\frac{1}{\sqrt{T}} Q \Rightarrow \xi V \tag{7}$$

where, Q is R/S statistic, T is the total number of observations in a full sample, ξ xi is a factor for adjusting short-range dependence.

The estimation procedure for ξ is as follows. First, assumes an series follows a stationary AR(1) process as eq.(8).

$$x_t = \rho x_{t-1} + \varepsilon_t, \tag{8}$$

where, $|\rho| \in (0, 1)$, $\varepsilon_t \sim w.n(0, \sigma^2)$.

In this case, the limiting distribution of $Q/\sqrt{T}$ will be ξV, where ξ is defined as follows :

6) Peters(1994) suggested a different approach for detecting LM in time series. The method calculates the Hurst exponent by classical R/S analysis after removing a short-range dependence by estimating a AR process. Our study would compared these two approaches.

$$\xi \equiv \sqrt{\frac{1+\rho}{1-\rho}} \tag{9}$$

As the distribution table of ξV is reported in his paper, it is possible to test a formal hypothesis of the non-existence of the LM. However, it is no trivial task to determine ξ if short-range dependence differs from AR(1) form.

Lo(1991) proposed another way to determine ξ in case a short-range dependence follows an ARMA(p,q) process.[7)]

$$\xi \equiv \frac{\lim_{T\to\infty} E\left[\frac{1}{T}\left(\sum_{j=1}^{T} \varepsilon_j\right)^2\right]}{\lim_{T\to\infty} E\left[\frac{1}{T}\left(\sum_{j=1}^{T} \varepsilon_j^2\right)\right]} \tag{10}$$

where, ε_j is the residuals of ARMA(p,q) model.

In practice, there are two problems in implementing R/S analysis or modified R/S analysis.

First, with respect to the number of data observations. It is a prerequisite to divide the full sample by the same size of subsamples. As mentioned earlier, the Hurst exponent could be obtained by OLS. Yet, a simple regression technique requires enough observations to get a desirable estimator. In other words, to obtain a significant unbiased estimator of the Hurst exponent, the number of divisions of the full sample by subsample should be sufficiently large.[8)] For example, given about 20 observations to get a significant estimate, the ways of dividing the full sample into subsamples should at least be above 20.

Second, R/S analysis requires that a sample period should be long enough for significant results since the aim of this method is to test the existence of LM in time series. However, the length of a current sample period is not enough for significant results. As it is not long since the Korean currency market launched a flexible exrate system, it may not be possible to get a significant result for the LM test.

In addition, the modified R/S analysis has a problem that it shows a low power for detecting some LM structures. The reason could be inferred from the fact

7) For details, see Lo(1991).

8) For example, if full sample had 4800 observations, the number of divisions of the full sample by same size subsample (k) would total 42. But Peters(1994) noted that if observations (N) in each subsample are smaller than 9, then it might result in a insignificant estimator. Also, in that each subsample should exceed at least 2 observations, only 34 subsample could be available to calculate the Hurst exponent. Despite being inadequate, the estimation by OLS may be possible.

that a pseudo-random number generator used for simulating the modified R/S test statistics is similar to deterministic chaotic structures.

3. Fractional differencing parameter estimation : GPH method

As above, the classical R/S and the modified R/S statistics were reviewed as detecting methods for intrinsic long memory (LM) property. This section will describe the fractional difference method based on the ARFIMA model. The method was developed for identifying a long-range dependence (LRD) of shock.

For modeling's aspect, ARFIMA(p,d,q) can capture a slowly decaying ACF. This approach is to test whether the difference parameter (d) is 0 or not. If $d = 0$, ARFIMA returns to ARIMA(p,0,q) model which has only short memory. But, if $d \neq 0$, these processes are said to have LRD.

Two methods can be implemented to estimate the difference parameter, d ; one is semi-parametric such as GPH (spectral regression), Robinson method, and the other is to utilize a joint MLE (Maximum Likelihood Estimation) in frequency domain such as Fox-Taqqu or Sowell method.

The Joint MLE type methods maximize the joint likelihood function of parameter d and other parameters (p,q). These methods are known to present more accurate estimation results than semi-parametric methods where model specification is known.

Here, we explain GPH, two-step estimation method, based on a simple regression of periodogram to estimate a difference parameter. The first step is to estimate d exploiting a spectral density function at near 0 frequency domain. The second is to estimate p and q in the ARMA model, which has been constructed transforming time series data from long memory filter with previously estimated d.

In this context, as our main interest is to identify the existence of LRD in time series, we will explain only the first step that estimates a difference parameter. The second step can be implemented simply by applying the common estimation technique for the ARMA model.

Considering the following time series with a stationarity after kth differencing,

$$X_t = (1 - L)^k Y_t \tag{11}$$

Next, estimate δ in the following equation[9)]

$$(1 - \mathrm{L})^{\delta} X_t = B(L)\, \xi_t \equiv u_t \tag{12}$$

9) Suppose Y_t is stationary, it is not necessary to do kth differencing.

Note that the difference parameter (d) of original time series Y_t is equal to $\delta + k$. If $\delta = 0$ and $k = 1$, it means that Y_t has a unit root, i.e., $d = 1$.

With $d = 1$, a spectral density function of X_t is as follows :

$$f_X(\lambda) = |1 - \exp(-i\lambda)|^{-2\delta} f_u(\lambda) = [4\sin^2(\lambda/2)]^{-\delta} f_u(\lambda) \tag{13}$$

where, $f_u(\lambda)$ is a spectral density function of stationary u_t.

Write jth Fourier coordinate in sample size of T as follows.

$$\lambda_j = \frac{2\pi j}{T}$$

where, $j = 1, 2, ..., m$, $0 < m < T$.

Taking log transformation of eq.(13), and then, add and subtract $\ln\{f_u(0)\}$.

$$\ln\{f_X(\lambda)\} = \ln\{f_u(0)\} - \delta \ln\{4\sin^2(\lambda/2)\} + \ln\{f_u(\lambda)/f_u(0)\} \tag{14}$$

We can represent eq.(14) as a simple linear regression form.

$$\ln\{I(\lambda_j)\} = \beta_0 + \beta_1 \ln\{4\sin^2(\lambda_j/2)\} + \eta_j \tag{15}$$

where, β_0 corresponding toln $\ln\{f_u(0)\}$ of eq.(14) is constant, $\eta_j \sim iid(0, \sigma^2)$.[10)]
Thus, we will get a estimate of $\delta$by estimating eq.(15) using $\beta_1 = -\delta$
A careful understanding is required of the fact that the above regression model can make sense only at a near 0 frequency.
In the above regression, it is known that an OLS estimate of β_1 is a consistent estimator for δ and it asymptotically follows a normal distribution.[11)] That is, the difference parameter, d, can be obtained by regressing the log periodogram on $\ln\{4\sin^2(\lambda_j/2)\}$ near 0 frequency in eq.(15). According to these results, this process only has short memory, if the hypothesis of $\delta = 0$ cannot be rejected. Therefore, we can provide a proper model specification for the series as ARIMA(p,1,q).
In summary, the estimation of ARFIMA(p,d,q) using the GPH method has the

10) Here, it should be noted that η_j etaj was not a last term, $\ln\{f_u(\lambda)/f_u(0)\}$, in eq.(14).
If considering a low frequency near 0, the term could be ignored as small. Therefore, adding $\ln\{I(\lambda_j)\}$ in both sides of eq.(14), and then rearranging, we get a following equation. $\ln\{I(\lambda_j)\} = \ln\{f_u(0)\} - \delta\ln\{4\sin^2(\lambda_j/2)\} + \ln\{I(\lambda_j)/f_X(\lambda_j)\}$, so, η_j indicates $\ln\{I(\lambda_j)/f_X(\lambda_j)\}$ in above eq.
11) For more in details, see Robinson(1992).

following procedure. The estimation of d to measure the degree of LRD is first step, and then, AR parameter (p) and MA parameter (q) are estimated, which belong to a short memory. This method has strength that it can flexibly capture the features of time series. Further, as the LRD can be represented by fractional difference parameter (d), not by time lags p and q, it is not necessary to lengthen p and q. In this sense, the ARFIMA model abides by so-called parsimony principle.

Ⅳ. Empirical results

Our empirical study employed the following sample data sets of Korean won/dollar exchange rates for identifying LM/LRD properties.

- Sample period : 3, Jan., 1992 ~ 15, Sept., 2000.[12)]
- Data frequency : daily (2502), weekly (501).[13)]

In addition to exchange rate itself, we estimate it because true volatility can not really be observed. in order to look for the existence of LM/LRD in exrate volatility. Therefore, for the estimated series by three models such as historical volatility, EWMA, GARCH model, the existence of LM/LRD are examined.

1. Exchange rate data

(1) Basic characteristics of won/dollar exrate

① Normality test

We begin with a normality test to check distributional characteristics of exrate series. As for the distribution of daily exchange rates, many previous studies reported it had fat-tails and kurtosis higher than 3 implied by the normal distribution. The distribution with such features is called as “leptokurtic” distribution. The leptokurtic distribution can be considered as a symptom of

12) The reasons for setting Jan. 1992 as the starting point of the sample period are as follows. In principle, a long-run sample period should be used for LM/LRD tests. Though far from perfect, the Korean currency market operated a form of a flexible exrate system since March, 1990. However, we think some preparing period may be needed for the new system, so the sample period runs from 1992. (Strictly speaking, a “true” flexible exrate system was only actually launched after the 1997 crisis.)

13) Each has observations of 2502 (daily), and 501 (weekly), respectively.

Figure 1 Comparison of distributional forms : Actual Exrate vs. Normal Exrate

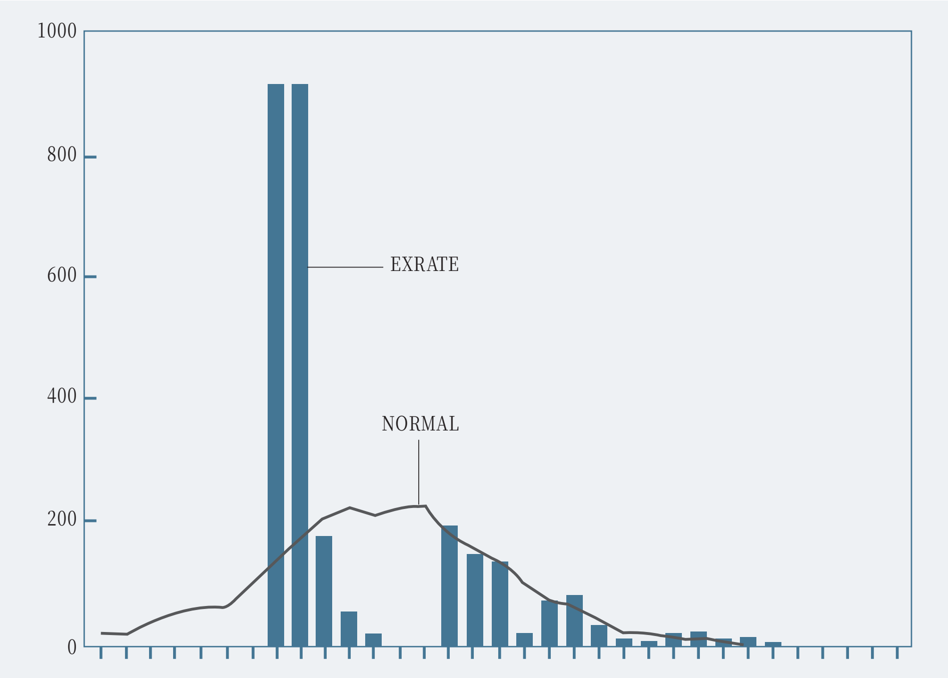


Table 3 Descriptive statistics

	Raw	Log difference (daily)	Log difference (weekly)
Mean	945.2242	0.00015	0.0007
S.D	229.6452	0.0102	0.0178
Skewness	1.3724	0.1497	5.9084
Kurtosis	4.0597	159.7316	84.3972
Jarque-Bera's Q-statistic	902.2228	2557818.0	140940.7

nonlinear structure. Also, the series with LM/LRD structures are expected to display such distributional characteristics, as the likelihood of extreme observations is higher.

Following [Figure 1] illustrates the distribution of actual exchange rates compared with the normal distribution, which is obtained from a pseudo-random generator. As shown, the actual distribution appears differently from normality.

Next, [Table 3] reports the descriptive statistics of exrate level, *1*st log differenced data (daily, weekly). All transformed data strongly reject the hypothesis of normality since Jarque-Bera's Q-statistic is very high.[14)]

② The structure of ACF

The existence of unit root must be clarified before estimating the difference parameter (d) of ARFIMA model. Once the series turns out to have unit roots, it should be differenced to recover stationarity. However, R/S analysis and the estimation of d were carried out in level data, which has a unit root, to understand the relationship between a unit root and the LM/LRD structures.

As shown in [Table 4], the daily exrate of won/dollar has a unit root, and thus it can be recovered stationarity through 1st differencing, i.e., it is a I(1) process.

Table 4 **Unit root test & ACF**

	Raw	Log difference (daily)	Log difference (weekly)
Unit root test (ADF)	-1.2913	-34.1756	-6.9191
Existing unit root	O	X	X
ACF	slowly decaying ACF	unclear	unclear

*Critical values in ADF test are as follows
- Raw & Log differenced data : -2.567(10%), -2.863(5%), -3.436(1%)
- Weekly : -2.570(10%), -2.868(5%), -3.446(1%)

③ ARCH test

According to [Figure 2], the magnitudes of variance between volatile periods and calm periods are clearly distinguished. This feature implies that the series has a time-varying volatility, which is called the "ARCH effect" or "Volatility clustering".

The standard LM test for the ARCH effect is applied to all data forms. The test results indicates the hypothesis of the non-existence of an ARCH effect is strongly rejected at the 1 percent significance level in all data forms. However, the degree of the ARCH effect is weakened in weekly log differenced data.[15)]

14) Jarque-Bera's Q-statistic is designed for testing the hypothesis of normality, it measures the differences of skewness and kurtosis from the normal distribution as following equation.

$$Q = \frac{N-k}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right),$$

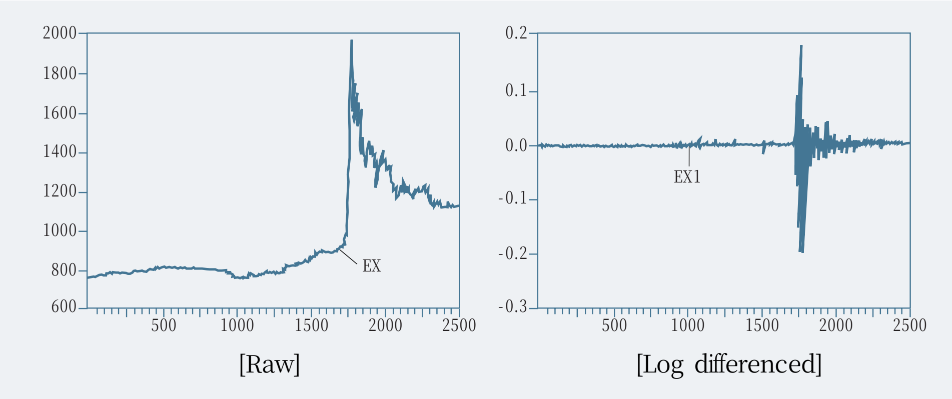
where, S : skewness, K : kurtosis, k : # of parameters.

Note that the Q-statistic follows a $x^2(2)$ - distribution under the hypothesis of normality.

15) Calculated LM test statistics are as follows.
- LM statistic in raw : 61.0274 (0.000)
- LM statistic in log differenced data (daily) : 125.5253 (0.0000)
- LM statistic in log differenced data (weekly) : 8.6878 (0.003)

* p-values in parenthesis.

Figure 2 Time series profile of won/dollar exrate



(2) R/S analysis

① Hurst exponent

The estimate of the Hurst exponent is 0.9379, close to 1, in the daily raw data, and had a unit root according to the results from the ADF test. The estimated H exponents in the log differenced data and the residuals of AR(1) filter are similar as 0.6401, 0.6356 respectively. These values are smaller than in the raw data.

These results are consistent with Lo(1991)'s view that H is likely to have an upward bias in the existence of short-range dependence. But, the modified R/S analysis results in the acceptance of the hypothesis of non-existence of LM in the daily log differenced data. These results will be reviewed below in more details.

The R/S analysis shows H=0.6318 for the residuals of AR(1)-GARCH(1.1) filter. This estimate is similar to the residuals of AR(1) filter. From this, we conclude that the GARCH structure does not have a serious impact on the LM property.

The estimated H in weekly data is 0.7545, which is higher than that H from daily data. This result is consistent with Peters(1994)'s view that the daily data shows less persistent trends than weekly or monthly data due to noise.

As the estimated H s for all data forms are larger than 0.5, the correlation measure (C), as explained earlier, would be positive values. It implies that the data has a positive feedback mechanism. This result indicates that won/dollar exrates follow a Biased Random Walk (BRW) rather than a random walk.

From the calculated C, we can expect that positive (or negative) price movements at some dates are followed by positive (or negative) price movements In the raw data with a unit root, the measured C is 0.835, which means a relatively high correlation.

In the log-differenced daily data and the residuals of AR(1) filter, the measured C s are 0.214 and 0.236, respectively. Though decreased, they show significant positive correlations.

Also, the measured C being 0.423 for weekly data, we can infer that the possibility of positive price movements is above 40 percent.

② V-statistic

Given that the analyzed data had an LM structure, we can expect the existence of a disappearing point of LM property. Accordingly, Peters(1991, 1994) emphasized that the point can be identified through a V-statistic graph.

V-statistic can be defined as follows.

$V_N = (R/S)_N / \sqrt{N}$

where, N : the number of observations.

If R/S statistic increases proportionally to the square root of time, the V-ratio will be constant. As such, log(R/S) vs. log(N) graph for a random process will be flat.

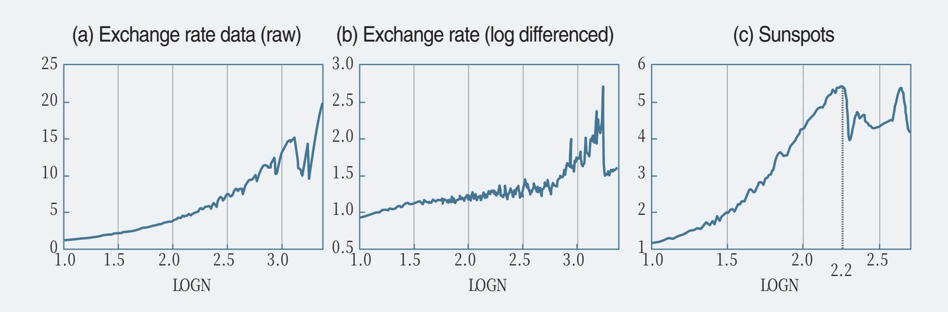
Suppose $H > 0.5$ and the R/S statistic increases faster than the square root of time, the graph would be a upward sloping one. If $H < 0.5$, the graph would be a downward sloping.

The series with a persistent trend will be a upward sloping initially, and then will flatten at the point of disappearance of the LM property. According to Peter(1994), the distance from initial to flattening point could be considered as a non-periodic cycle.

However, the empirical results of this study on the existence of non-periodic cycles in all transformed exchange rate data show that most data do not have a clear breaking point on the V-statistic graph. Therefore, it is uncertain whether won/dollar exrates have non-periodic cycles.

Strictly speaking, though the raw data shows the breaking point as approximately $\log(N) = 3.1$, it is difficult to refer to it as a nonperiodic cycle since it shows several irregular behaviors. Log differenced data (returns) show more irregular pattern than the raw data, so it is impossible to identify the breaking point.[16)]

Figure 3 Graph of V-statistic



But the estimation result of sunspots' monthly data (1947~1991), which are known for having non-periodic cycles, show a clear breaking point at about $\log(N) = 2.2$. Hence, it is possible to identify the non-periodic cycle as about 11 year.[17)]

The discussed non-periodic cycles cannot be detected by the spectrum analysis that was used to find periodic behaviors in time series analysis.

③ Modified R/S analysis

As explained above, the modified R/S analysis provided by Lo(1991) is to analyze long memory after modifying short-run dependence in the classical R/S statistic. The modified R/S statistic from our sample data indicates that the hypothesis of non-existence of LM cannot be rejected at the 10 percent significance level. The results of weekly data, however, reject the hypothesis at the 10 percent significance level.

This result is in-line with the result of R/S analysis for AR(1) residuals, which supported the existence of LM structure after removing short-run dependence. However, as mentioned by Lo(1991), the modified R/S analysis did not have high power for some LM structures. Hence, if the series had a nonlinear structure such as chaos, the results of the R/S analysis cannot be reliable.

To conduct a formal test on this inference, we applied the modified R/S

16) Specifically, the log differenced data showed a steepening slope over the log(N). This result could not be consistent with Peters(1994)'s view. Peters explained V-statistic showed a upward sloping at starting point, and then started flattening at the point of LM disappearing. However, the results of this study are almost consistent with Peters(1994)'s results in that nonperiodic cycles cannot be identified in the time series of major currencies.

17) R/S analysis on sunspots data resulted in having a strong persistent trend as H is 0.7810. Clear evidence for a nonperiodic cycle could not be found in our empirical study. This result was likely produced from the large amount of noise embedded in high-frequency exrate series, so the analysis for nonperiodic cycles required long-run low frequency series.

statistics for the series generated by the tent map, a deterministic chaotic function. The hypothesis of not having LM structures cannot rejected at the 10 percent significance level, so we can conclude some nonlinear structures, for example, chaos, which is not detected by the modified R/S analysis, exist in the won/dollar exchange rates.

(3) The estimation of difference parameter (d)

In using the 2-step estimation procedure of GPH method here, since our interest is limited to the fractional differencing parameter (d), we will process 1-step only.

As the estimated d is higher than 0.5 in the raw data, though it is not contrasted with the result of unit root test, I(1) process, it is likely to create a loss of information due to 1st differencing. The integer differencing method for stationarity is too restrictive.

Mostly, the estimated d s were similar to the size with the theoretical relation of $H = d + 0.5$. But there emerge negative values for the log differenced daily data and AR(1) residuals, thus it is not consistent with the result of R/S analysis which showed these series were persistent processes.

However, these results are not credible due to a high standard error. The theoretical relation between H and d, i.e., $H = d + 0.5$, is not clearly binding in weekly data. The reason is that the sample of weekly data is too small. (observations : 500).

[Table 5] summarizes the empirical results for LM/LRD in the won/dollar exrates by usage of the R/S, the modified R/S analysis, and the estimation of d.

Table 5 **Empirical results of LM/LRD**

	Raw	Log - differenced (daily)	AR(1) residuals	Log - differenced (weekly)
H exponent (standard error)	0.9379 (0.0033)	0.6401 (0.0029)	0.6356 (0.0028)	0.7545 (0.0069)
Modified R/S H_0 : No LM	accept (at 10%)	as left	as left	Accept (at 5%) Reject (at 10%)
d (standard error)	0.5275 (0.0211)	-0.0109 (0.0145)	-0.0680 (0.0149)	0.4216 (0.0446)
C (correlation measure)	0.835	0.214	0.206	0.423

*standard error in parenthesis

(4) Other issues related to LM/LRD

① Previous empirical results

Perron(1989)'s remark that the existence of unit root could be confused with the break of trend in the vortex of unit root revolution needs to be reconsidered in the current contexts.

Structural breaks invoking big changes in an economic system could be related directly or indirectly to the properties of LM/LRD. Intuitively, given that the long-run impact of a shock cannot persist perpetually, we can infer the disappearance point of shock's effect as the point of regime switching.

Many studies on the relationship between LRD and the structural breaks/regime switching have already been performed, however, most of the literature treated these two properties independently and separately.

Diebold-Inowe(1999) insisted that these two properties had a close relationship. They illustrated through simulations that stochastic regime switching was often confused with the LRD. Specifically, this study generated simulated data by using some stochastic break models, for example, the Markov-switching model of Hamilton(1989), and the stochastic permanent break model of Engle-Smith(1999). And then, they estimated a differencing parameter (d) by the GPH method for the generated data.

The estimation results illustrated the estimated d ranged between 0 and 1, thus, the series generated by stochastic break models may be confused with the series with LRD structures.

Lobato-Savin(1997) had estimated d on S&P 500 returns.

Though the estimation results of d did not support that the returns had the LRD, the nonstationarity and aggregation in returns' absolute value (2nd moment) could generate the LRD feature shown as the fractional value of d.

Similar to Granger(1980)'s comments, they empirically illustrated that the LRD not observed in the second moment of individual stock can be generated in the aggregation process for index.

Further, as the second moment displayed time-varying property and volatility clustering, we estimated the d in each of several subsamples to check the possibility that the nonstationarity can generate the LRD property. Each stationary subsample showed the LRD, therefore, the hypothesis that nonstationarity can generate the LRD was rejected.

Moreover, Gourieroux-Jasiak(2001) indicated that infrequent structural breaks can be confused with the LRD as result of the estimation of ACF. However, they showed that ACF in nonlinear dynamics with regime switching, though not in traditional linear models, can be confused with the LRD.

On the other hand, there exist some studies which did not agree the idea that regime switching structures could be separated with the LRD. Timmermann(1999) pointed out that a standard Markov-switching model displayed high persistence during a short run period in spite of I(0). Also, Ryden, Terasvirta, and Asbrink(1998) insisted the Markov-switching models cannot represent the LRD structure properly.

Diebold-Inowe(1999) also refrained from drawing the conclusion that the structural breaks can generate a spurious LRD phenomenon. Although the actual series are generated by the regime switching structures, LM/LRD can still be useful properties. The fact that the data with the LM/LRD implies the existence of intermittency indicates the disappearing point of such effects.

Hence, the prediction models for these data need to be constructed on the basis of regime switching schemes. The algorithm considering the LM/LRD property and the structural breaks simultaneously is required for prediction of the interested series.

In addition, as pointed out by Granger(1998), the relation between a stationary process with regime switching structures and the LM/LRD is still unclear. Although the two approaches seem to be similar, no clear relationship is known so far. However, we can infer that the structural breaks and the LM/LRD are two different views of the same phenomenon. Therefore, it is not reasonable to insist one is true, another is not.

Finally, we require more delicate theories for explaining the relationship between the LM/LRD and the structural breaks often observed in economic and financial time series.

② The estimation of d in subsamples

As shown, it is possible that the LRD properties observed in time series are actually generated by structural changes. Moreover, the data employed in this study includes the “Currency crisis” period, which can be regarded as a structural change.[18)]

Therefore, we divided the full sample into two subsamples, and used the year 1998 as the starting point of second subsample. According to Lobato-Savin(1997), we estimated a difference parameter in each subsample.

If the estimated d s differ significantly from the estimated d in the full sample, it can be deduced that the LRD is likely to have occurred by structural change.

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18) Thanks are due to an anonymous referee for bringing up this point.

Table 6 Estimation results for d in subsamples

	Sample period	Estimated d	Reference
Before Asian crisis	1992.1.3 1996.12.31	0.5859 (0.029)	0.5275 (0.0211) in full sample
After Asian crisis	1998.4.1 2000.9.15	0.5810 (0.039)	

*Standard error in parenthesis

Otherwise, the DGP of the exchange rates series might have an LRD structure originally.

For testing this hypothesis, we separated the full sample of won/dollar exrates into two subsamples. One period before IMF bailout is from January 1, 1992 to December 31, 1996, the other period is from April 1, 1998 to September 15, 2000.

The GPH estimation results for d in the separated two subsamples are summarized in [Table 6].

Surprisingly, the results show that the estimated d s in both subsamples are higher than d in the full sample. From these results, we can conclude that the LRD property expressed as the fractional difference parameter does not stem from structural breaks.

The empirical test for links between LRD and stochastic regime switching considered in Diebold et al.(1999)'s work might help us to arrive at more concrete conclusions.

2. The LM/LRD of exchange rate volatility

Concerning the analysis of the volatility of financial variables, the main difficulty is in setting up an appropriate proxy variable since true volatility is not observable. Hence, we need to decide the proxy series of the volatility, and then estimate the existence of the LM/LRD properties in the volatility series.

Here, we will give rise to the proxy series of won/dollar exrates volatility by using three models ; namely, Historical volatility, EWMA, and GARCH. These models are considered as the most popular volatility models in financial time series analysis.[19]

19) In addition to these three models in our study, the realized volatility model suggested by Anderson-Bollerslev(2002) etc. The method is mostly used to measure volatility in the high-frequency data.

(1) Historical Volatility

We refer to asset price as y_t, and write tilde $\tilde{y}_t$ as deviation from a sample mean. Also, we can regard $z_t = \tilde{y}_t^2$ as the proxy variable for a sample variance.

The historical volatility (HV), n-period moving average for period T, can be calculated as follows.

$$\hat{\sigma}_T^2 = \frac{1}{n} \sum_{t'=T-n}^{t'=T-1} z_t \tag{16}$$

where, the moving average period (n) is set to be 30.

The result of ADF test for HV indicated the non-existence of a unit root, which indicates the series is a stationary process, I(0). However, it displayed a very slowly decaying ACF in contrast with a random process.

The daily log differenced data indicated the ACF would be 0.002 at a lag of 36, however, the calculated ACF at a lag of 36 in HV series was 0.533. Hence, the exrate volatility series measured by the HV model showed a strong LRD property.

The Hurst exponent was 0.8602, which was relatively high even though the series was I(0). But the difference parameter (d) in ARFIMA used by GPH was 0.7107, which could not satisfy the stationary condition ($|d| < 0.5$), showing different results from the fact produced by the unit root test.

We can conclude the HV series had a strong LM/LRD from the fact that the ACF decayed very slowly, both the Hurst exponent and difference parameter are larger than 0.5. Through a knife-edge distinction between I(0) and I(1) such as the unit root test, we cannot detect these features.

Further, we can realize the fact that the traditional view of ARIMA, which considers an I(0) process as a stationary and mean-reverting process, cannot explain a nonstationary and mean-reverting process ($0.5 < d < 1$)

(2) Exponential Weighted Moving Average : EWMA

The method of VaR calculation as RiskMetrics developed by JP Morgan propose using the EWMA model for measuring volatility. This is a volatility model that imposes more weight on recent variances relative to a simple moving average model.

We are able to calculate a n-period EWMA as follows.

$$EWMA = \frac{z_{t-1} + \lambda z_{t-2} + \cdots + \lambda^{n-1} z_{t-n}}{1 + \lambda + \lambda^2 + \cdots \lambda^{n-1}} \tag{17}$$

where, λ lambda is a relative weight ($0 < \lambda < 1$), and z_{t-k} ($k = 1, 2, \cdots, n$) are the past realized variances.

From eq.(17), we can figure out that the EWMA gives more weight to recent variances than past ones.

Asymptotically, considering $n \rightarrow \infty$ in eq.(17), we can get a the following equation.

$$EWMA = (1 - \lambda)\sum_{i=1}^{\infty} \lambda^{i-1} z_{t-i} \tag{18}$$

The EWMA in eq.(18) is equally regarded as the following IGARCH(1,1) model.[20]

$$h_t = \lambda h_{t-1} + (1 - \lambda) z_{t-1} \tag{19}$$

where, the coefficient of h_{t-1} and the coefficient of z_{t-1} is summed to 1, so the effect of volatility shock persists infinitely.

In summary, the basic idea of EWMA is that a smoothing constant λ is set to be smaller than 1, thus, more weight is imposed on recent variances. This method has the advantage of simple calculation while there remains the problem of specifying λ arbitrarily without theoretical foundations, as pointed out by Muth(1961). Nevertheless, implementing EWMA is relatively simple since we only estimate one parameter, λ

The estimated H exponent in the EWMA series calculated by eq.(17) is 0.6944, thus, it showed a significant persistent trend. The ACF at lag 36 is 0.121, which is a decrease compared to the one measured in HV, however, it still preserves a slowly decaying ACF relative to a random process.

The estimated d in ARFIMA is 0.6417, which means that the series is nonstationary. This result is not consistent with the theoretical relationship between H and d.

(3) AR(1)-GARCH(1,1) model [21]

To estimate a conditional variance of won/dollar exrate data, we used the

20) Reiterating h_{t-1} in eq.(19), we can get the EWMA form in eq.(18).

21) The reasons for estimating the volatility by AR(1)-GARCH(1,1) model are as follows. First, setting 1 for the AR time lag in the mean equation resulted from the Wald test, where the coefficients of time lag 2 and lag 3 were not significant. And as the correlogram illustrated that, after time lag 1, the ACF did not display a distinctive feature, we selected the AR(1) model specification as a mean equation. Second, for modeling a conditional heteroskedasticity, GARCH(1,1) model is most popular in the field of financial time series analysis. Naturally, if the volatility showed a strong LRD, the FIGARCH model should be used to estimate the true volatility.

Table 7 The test results of LM/LRD on exrate volatility

	HV	EWMA	AR(1)-GARCH(1,1)
R/S analysis (Hurst exponent)	0.8602	0.6944	0.7577
d in ARFIMA (standard error)	0.7107 (0.0223)	0.6417 (0.0279)	0.3997 (0.0194)
ACF	very slowly decaying (lag 36 = 0.533)	slowly decaying (lag 36 = 0.121)	slowly decaying (lag 36 = 0.099)
Unit root test	Reject H_0	Reject H_0	Reject H_0

* Note : H_0 is the null hypothesis of the existence of unit root (The significance level is 0.05)

following model.

$$r_t = \alpha + \beta r_{t-1} + \varepsilon_t$$
$$\varepsilon_t / \Omega_{t-1} \sim N(0, h_t)$$
$$h_t = c + \gamma \varepsilon^2_{t-1} + \delta h_{t-1}$$

The result of the R/S analysis on the volatility series measured by the above model is $H = 0.7577$, which means the exrate volatility has a stronger LM property than the exchange rates level itself.

If the exrate volatility increases at a certain point for any reason, the increased volatility would remain for some period due to market instability, which is called “volatility clustering”, a common feature in financial markets.

As the estimated *d* by GPH method is 0.3913, this implies that the series is stationary. This result is different from volatility series HV as well as EWMA, therefore, we can infer that conditional heteroskedasticity was able to explain a large portion of the nonstationarity that emerged in a second moment. However, as the estimated *d* was larger than 0, it still showed a high persistence of shock.

As shown in [Table 7], all the R/S analysis on the series made by three volatility models resulted in a strong LM property, even though after adjusting for a short-range dependence. The estimated *d* in ARFIMA also illustrated that the exrate volatility had a very persistent trend.

Therefore, we can conclude that the modeling of exrate volatility requires FIGARCH type rather than a standard GARCH model because FIGARCH is able to capture the long memory property. Furthermore, FIGARCH specification is parsimonious like ARFIMA in the sense of reducing the number of parameters.

However, it is difficult to reach a clear conclusion on stationarity through the estimated *d* value, as the estimated *d* s differ between GARCH and other models (HV, EWMA).

Ⅴ. Conclusions

This study tested for the existence of LM/LRD structures in the Korean won/dollar exchange rates and volatility. We conducted the R/S analysis and the modified R/S analysis to test the LM property, and estimated *d* in ARFIMA, a generalized form of ARIMA to measure the degree of shock's persistence.

The results of estimating the Hurst exponent and *d* rejected the hypothesis that long memory properties did not exist in exrates series.

Considering the possibility that the structural changes may generate long memory properties observed in financial time series, we divided the full sample into two subsamples ; before and after the Asian crisis. And then we estimated *d* respectively in each subsamples.

As the existence of LRD was not rejected, the estimation results of the subsamples were not different from the full sample. This coincided with Batten-Ellis(1996) who demonstrated the LRD property in yen/dollar exrate series by spectrum analysis.

Our result supported Kim(1997)'s empirical findings that the won/dollar exrate had positive autocorrelations, and did not have a mean-reverting property to Purchasing Power Parity (PPP) level for short-run periods.

According to the modified R/S statistic, we could not reject the null hypothesis (non-existence of LM) at the 5 percent significance level. However, this result was likely to emerge from the fact that the modified R/S analysis had a low power on some nonlinear structures. If the LM or LRD property exists in exrate time series, there are very important implications for currency market analysis.[22)]

The implications can be summarized as follows.

First, it is difficult to identify the parity condition of forward exrate due to the existence of long memory structure.[23)]

22) Even without the empirical results of R/S analysis or d estimation, we can infer the long memory property in exrate series for a common feature such as the bandwagon effect frequently observed in currency markets.

23) As an anonymous referee pointed out, despite the exrate series having a long memory property, it could not be taken adduced directly to deny the parity condition of forward rates. Simply, if the estimated d approaches 1, the mean-reverting speed to parity level was very slow, so the traditional methodologies could not detect such features.

The previous studies suggested that despite spot rates and forward rates being I(1), there did not exist a so-called cointegrated relation. However, Baillie-Bollerslev(1994) claimed that the forward premia could be well represented by the ARFIMA(2,d,0) model for monthly dollar based exchange rate series.

Therefore, without referring to the cointegrated relation as a common CI(1,1) paradigm, a long-run equilibrium relation between spot rates and forward rates could be discovered by a fractional cointegration approach. Reasonably, fractional cointegration will illustrate a slower mean-reversion than the standard cointegration.

The tests for LM/LRD (such as R/S analysis and the estimation of d) have the advantage that the degree of memory can be figured out through H and d, which helps to measure the speed of mean-reversion.

Further, despite the process with $0.5 < d < 1$ being nonstationary, as it shows mean-reversion, we can separate the stationarity from mean-reversion property. This issue cannot be understood under the traditional I(0)/I(1) paradigm.

Second, long memory properties can provide new insights into the debate on the validity of the Purchasing Power Parity (PPP) theory. Many researchers have worked to test whether the long-run PPP might exist since the introduction of the flexible exchange rates system in 1973. A lot of papers arrived at negative conclusions regarding the validity of PPP. However, the empirical works were mainly processed on the basis of I(0)/I(1) paradigm, thus, only limited results on the long-run equilibrium relation could be provided.

Diebold *et al*.(1991) estimated ARFIMA(p,d,q) model by the Fox-Taqqu method, the results of which illustrated that the process became mean-reverting ultimately, though the effect of the shock to equilibrium persisted for a long time. Therefore, he strongly claimed that the long-run PPP could materialize. Also, Cheung-Lai(1993) found that a fractional cointegrated relation between nominal exchange rates and relative prices had been binding.

The traditional theories on the determination of exchange rates considered that the equilibrium level of exrates could be determined by fundamentals such as relative prices, interest rates etc. But suppose the relation between the exchange rates and fundamentals is fractionally cointegrated, the deviations from the equilibrium level may remain for a long period. These deviations may persist due to noise trading or market sentiment, for example herding behavior, as pointed out by Keynes.

Thus, the persistence will disappear and the exchange rates return to the equilibrium level if some factors which can correct the biased prediction of noise traders or market sentiment are observed.[24)]

Third, asset returns cannot be specified as a linear model like a random walk since they do not follow a normal distribution.

The distribution of exrate series may be fat-tails, i.e., leptokurtic distribution, where extreme values occur frequently relative to normal pattern.

This suggests that the $T^{1/2}$-rule used for multi-period expansion of volatility or VaR (Value-at-Risk) may lead to over/underestimation of true volatility. The rule is valid only if underlying asset prices follow a normal distribution. However, the rule is commonly used especially in the field of option and risk theory. Thus, while true prices do not follow normality, applying the $T^{1/2}$-rule may cause mispricing of derivatives and financial risk.

The implications for trading strategies under the existence of long memory property will be different from that under Efficient Market Hypothesis (EMH). According to EMH, new information entering the markets should be reflected in prices instantaneously. If long memory exists, however, prices reflect new information over a long period, which means the market is not efficient.

In other words, it is possible to construct prediction rules based on past prices for markets with a long memory structure, while predictions based on the past history of prices could not allow excess returns under EMH.

For example, suppose the won/dollar exchange rates series with a long memory property were to be revalued at a certain date, then it can be expected the property would persist for a long time. Therefore, there exists an arbitrage opportunity only by keeping long positions in that revalued currency (won) until the long memory property disappears.

In the case of anti-persistent processes ($d < 0$), as frequent reversals of trend occur, active trading strategies that often switch between long and short positions may be appropriate.

In addition to exchange rates, other financial variables can be reviewed for long memory properties. For example, Kim(1994) empirically tested if a long-run Fisher effect exists for Korean interest rate series by using the fractional differencing method.

He reported that a long-run Fisher effect was shown to exist as nominal interest rates and inflation rates had a one-to-one correspondence relation, which meant real interest rates displayed a mean-reverting tendency.

Concerning this issue, we need to review Tsay-Chung(2000)'s work to show that the result is stemmed from OLS estimation technique. They illustrated that a spurious result of OLS analysis can arisen from the long memory properties, not

24) Peters(1991) noted that it is proper to represent asset prices determined by fundamentals as a range rather than a specific value. Therefore, the persistent trend could be shifted to a new range as the arrival of new information.

by nonstationarity.

According to Tsay-Chung(2000), suppose the sum of dependent variable's *d* and independent variable's *d* is larger than 0.5, there occurs a spurious effect in OLS analysis. Therefore, to get a significant result, firstly, the difference parameter (*d*) of variables should be estimated.[25)]

Next, the R/S results on exrate volatility provided that the estimated *H* ranged from 0.6944 to 0.8602, which showed higher values than exchange rates itself. The strong LM in the second moment suggested that the FIGARCH specification could be more appropriate than the standard GARCH types.

FIGARCH(p,d,q) is more efficient since it does not require a high value of *p* and *q*, as the LRD property can be mainly represented by noninteger *d*, which coincides with so-called “parsimony principle”. Moreover, if the DGP of the won/dollar exrates actually had a long memory structure, it was expected the FIGARCH model would have more accurate predictive power.

Finally, the important point concerning the long memory property is that such a property is highly likely to be related to a fractal structure.[26)] Thus, if the long memory structure were connected with fractals, it would not be appropriate to use ARCH types for volatility modeling, though these are popular in financial market analysis.

The reason is that ARCH types cannot represent fractal structures such as self-similarity and nonperiodic cycles, though the models can explain the portion of nonlinear dependence involving volatility clustering and fat-tails.[27)]

If exchange rates have a long memory property, there exists a certain point where the property disappears. The point can be considered as “intermittent structure”, thus, the prediction algorithm for exchange rates should be based on the state-transition concept considering such intermittency. Research on the prediction algorithm using fractal structures of exrates remains a task for further study.

25) Thus, if the sum of the difference parameters is larger than 0.5, we need to use a fractional cointegration technique rather than OLS.

26) Mathematically, fractals imply non-integer dimensions, and statistically, it indicates that time series have intermittent or inhomogeneous structures.

27) To detect such fractal structures in time series, it is necessary to estimate a correlation dimension, however, this goes beyond this paper's objections.

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